ISSN: 2278-3369
International Journal of Advances in Management and Economics
Available online at www.managementjournal.info

RESEARCH ARTICLE

Beyond Modigliani-Miller's Propositions

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Abstract

Modigliani-Miller's Propositions on business financing have become the usual paradigm on this subject: the main reason to prefer bonds and other liability instruments instead of shares consists of the "tax shield" provided by income tax savings on interests. The aim of this paper is providing a refutation for that concept and pointing out a basis for an alternative theory. A light refutation about tax shield may consist of the differences of taxation regimes in different countries and their subsequent changes. But this paper develops a strong refutation: tax shield is not the core variable in a general theory of financing.

Introduction

Modigliani-Miller´s Propositions (MMP) pertaining business financing have become the usual paradigm for this subject [1]. This is remarkable since they were first published more than fifty years ago [2]. Afterwards, the authors modified them stressing the importance of taxation in financing decision-making. They found that the main reason to prefer bonds and other liability instruments instead of shares consists of the "tax shield" provided by income tax savings on interests [3].The aim of this paper is providing a refutation for that concept and pointing out a basis for an alternative theoretical scheme for financing decision-making.

MM Propositions

The Propositions of Franco Modigliani and Merton Miller (MM) [4] is the generally accepted theoretical framework in the academic field. According to the same, funding decision is trivial to generate value in a company. The financing structure, dividend policy and any decision whatsoever in this regard is irrelevant to alter the performance of companies, for an efficient market makes instant arbitration of all yields in each business category. MM is based on information outside the company: the market prices. The market value of enterprise j is Vj and satisfies the following basic relationship:

$$\theta_k = \frac{\bar{X}_j}{V_j} \text{ where} \tag{1}$$

$\bar{X}_j$: likely performance of the company j within the activity k, θ_k : capital cost thereof. The company's market value is being available as the sum of the market value of shares A_j and debt

$$D_j : V_j = A_j + D_j = \frac{\bar{X}_j}{\rho_k} \tag{2}$$

The three meanings of the coefficient ρ_k for MM are as follows: (a) is the probable rate of return of stock j within business k, namely: $\rho_k = \frac{\bar{X}_j}{P_j}$ where the share price of j will be Pj, (b) is the price for financiers or investors for business k in the expression: $1 / \rho_k$, and (c) is the market capitalization rate for the expected value of a stream of benefits provided by its assets in activity k.
But with intellectual honesty, the authors warned that usual financial administration contradicts inferred consequences of this theory, since companies tend to devote resources and efforts to obtain competitive financing sources, beginning with liabilities.

To save their theoretical framework, they introduced an additional ad-hoc hypothesis: the role of income tax [5].The computation of interest on the debt as deductible cost for this tax base gives an advantage against the cost of equity, which is not deductible. This apparent tax savings to be generalized for a number of years tending to infinity got their third proposition on the Tax Shield that would increase the market value of companies.

In this second version, the formalization of MM Propositions is as follows

Proposition I: $$\boxed{\frac{\overline{X}_j}{V_j} = \theta_k - t\cdot(\theta_k - r)\cdot\frac{D_j}{V_j}} \quad (3)$$

Proposition II: $$\boxed{l_j = \theta_k + (\theta_k - r)\cdot\frac{D_j}{A_j}.(1-t)} \quad (4)$$

where:

l_j : probable rate of return on firm j

r : rate of borrowing costs

Proposition III: $$\boxed{VF = \frac{(t\cdot r\cdot D)}{r} = t\cdot D} \quad (5)$$

where:

VF : present value of Tax Shield for the deduction of interest on liabilities when the number of years tends to infinity

t : income tax rate

D :the company's debt

Refutation of Proposition III

Proposition I makes a full generalization of arbitraging. Every yield goes towards the same rate, given some level of risk, although in fact both internal and external factors may affect them.

However, Proposition II puts forward the difference between two rates: for net assets yield and for in debtness. The authors supposed a priori that the former is always higher than the latter.

This Proposition supposes that only case. But the latter may also be higher than or equal to the former in some phases of the business cycle.

Proposition III is based only on the tax factor. We shall show the basis for an alternative framework for a general theory of financing and shall refute the existence of above mentioned tax shield.

A light refutation may consist of the differences of taxation regimes in different countries and their subsequent changes. For instance, dividends, interests and similar concepts may be not taxed wholly or partially or there may be another sort of taxes affecting them (for developing areas, tax heavens, etc.).

But this paper provides a strong refutation. There is not such a thing as a tax shield as the central variable in a general theory of financing. Its effect may be evaluated only on a case-by-case basis for particular situations, as an auxiliary variable.

The new framework for a general theory of financing states that the main variables are operative EBITDA per assets on the one hand, and liabilities cost on the other hand. The first should be higher than the latter in order to increase net worth yield and therefore its market value on the long run.

The main rationale is that different taxation rates (even null) do not interfere with those central variables in determinating net worth yield. Therefore, tax shield is not a main variable for general theory of financing, but only one of the auxiliary variables that may be taken into account in particular cases, such as political influence, bankruptcy risks, economic environment, etc.

Hypothesis 1:

In a general theory of financing, the incidence of indebtness cost in cost of capital is not determined by the income tax factor, since income taxes influence the whole cost of capital.

Using traditional accountancy values, its algebraic expression is as follows:

$$\frac{E}{NW} = \left[\frac{EBIT}{(L+NW)} + \left\{\frac{EBIT}{(L+NW)} - \frac{I}{L}\right\}\cdot\frac{L}{NW}\right]\cdot(1-t)$$

(6)

where:

g : Profitability rate

E : Final net earnings

NW : Net worth

$EBIT$: Operative earnings before interests and income taxes

$k(i)$: Liability cost rate (including every associated cost)

L : Liabilities

A demonstration for this hypothesis is shown in Appendix 1.

This expression is similar to, but not equivalent to, Modigliani-Miller's Proposition II. The difference consists of their using of market values for the same concepts. The rationale of using accountancy values instead of market values is a difference between financing and investing decision making. Replacing in (6) accountancy values for market values, we obtain this formula:

$$l_j = \left[\theta_k + (\theta_k - r)\cdot\frac{D_j}{A_j}\right]\cdot(1-t) \qquad (7)$$

Whereas the variable equivalences are as follows:

l_j : Equivalent to $\frac{E}{NW}$ as per definition

θ_t : Equivalent to $\frac{EBIT}{(L+NW)}$ as per formula (2)

r : Equivalent to $\frac{I}{L}$ as per definition

$\frac{D_j}{A_j}$: Equivalent to $\frac{L}{NW}$ as per definition

It should be noted that expression (7) is different from usual Proposition II (4).

The difference consists of application of the tax factor, as per the algebraic demonstration based on usual definitions of net earnings as operative earnings minus financing costs and then minus income tax.

New Propositions

A new theoretical framework for financing decision-making ought to take into account two standpoints. An external standpoint follows the EMH. An internal one admits endogenous constraints in certain phases that may lead to material decisions towards financing optimization

Cost of capital may have two meanings. On the one hand, from the external standpoint it is the market rate suitable for investors for an economic activity *a priori*. On the **other** hand, it is the profitability to be maximized (subject to certain restrictions: long term sustainability, legal and ethical limitations, stakeholders interests, etc.) in order to obtain market value maximization *a posteriori*

This may cause vagueness or even redundancy, as shown in following hypothesis.

Hypothesis 2

A given relationship between interests and cost of capital is equivalent to that very relationship between interests and cost of net worth

For $k_{(0)}, k_{(i)}, k_{(e)} \geq 0$:

$$(k_{(i)} \leq k_{(e)}) \Leftrightarrow (k_{(i)} \leq k_{(0)})$$

$$(k_{(i)} \geq k_{(e)}) \Leftrightarrow (k_{(i)} \geq k_{(0)}) .$$

Demonstration is developed in *Appendix 2* (arriving to formulae B.3 and B.4).

There are real and perceived variables, as defined by Herbert A. Simon y James G. March [6]. Market arbitration tends to a homogeneous real cost of capital for every company within a given activity. However, decision makers are longing to achieve a higher profitability than the market median. How?

The main quantitative variable for financing theory should be the comparison between operative earnings (EBIT) on assets vis à vis cost of liabilities, or respective cash flows in the short run. Provided the former is higher than the latter, decision makers will achieve better profitability. Although financing decision-making may be a complex process including many variables, this is most important quantitative f**actor**.

The origin of dividends and interests are operative EBIT. Therefore, should be no EBIT, there will be neither interests nor dividends to distribute, nor income taxes to pay.

For decision-making, relative operative EBIT or ebitV should be taken into account, i.e. operative EBIT divided into assets that originate them, as follows:

$$Ebit\ V = EBIT\ /\ V \qquad (8)$$

where:

EBIT: operative earnings (before interest and income taxes)

V : operative assets

For short-term analysis, the same comparison may be made taking into account relative operative cash flow rate instead of relative operative profits, as follows:

$$EBITDA\ /\ V \qquad (9)$$

where:

EBITDA: operative cash flow (earnings before interests, income taxes, depreciation and amortization)

Using assumptions on general market equilibrium is usual in classic Economics theories pertaining interest rate and the financial market as a whole [7]. However, such generalizations may turn out to be too simplistic for business administration. Sometimes, it is possible to better off profitability with sound decisions on operative assets and its EBIT, and liabilities and its interest. This possibility takes place when the market is not completely efficient because of the kind of business or a special phase of economic cycle, which may cause some segmentation in different financial markets.

Modigliani-Miller as other authors suppose that every business can obtain liabilities at **a** cost $k_{(i)}$ that is lower than their cost of capital $k_{(o)}$.

However, in special cases as small companies, emerging markets, or some critical phases of the business cycle, financial conditions may change. Therefore, liabilities cost may turn out to be higher, lower or equal to operative EBIT on assets, with different final profitability on net worth. It is necessary to study these three possible cases.

***Case (1) Efficient and Stable Markets:* Operative EBIT Rate is Equal to Liability Cost**

There is a 1 to 1 relationship. There is a null incidence of financing on profitability whatever level of liabilities is used. This is similar to the Modigliani-Miller Propositions case.

Defining profitability as operative earning divided into net worth, and liability level as debt divided into net worth, the relationship may be shown in Exhibit 1.

Exhibit 1: Operative EBIT rate equal to liability cost

***Case (2) Coherent Downwards Market*: Operative EBIT rate is Lower than Liability Cost**

In this case, the higher liability level, the lower profitability on net worth. The latter diminishes quickly and may produce losses soon, as shown on *Exhibit 2*.

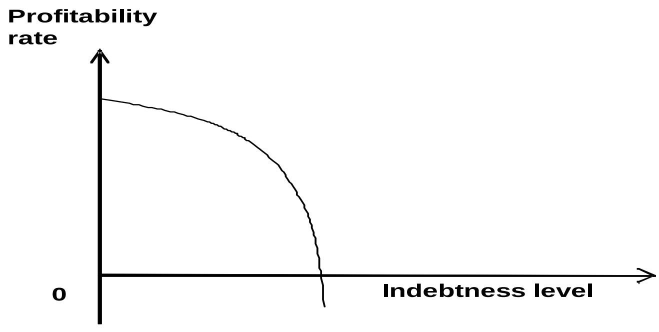

Exhibit 2: Operative EBIT rate lower than liability cost

***Case (3) Coherent upwards market*: Operative EBIT rate is higher than liability cost**

In this case, the higher liability level, the higher profitability on net worth. It should be put forward that the speed of this effect is lower than in the former case and is especially noticeable at high levels of liability, as shown in Exhibit *3*.

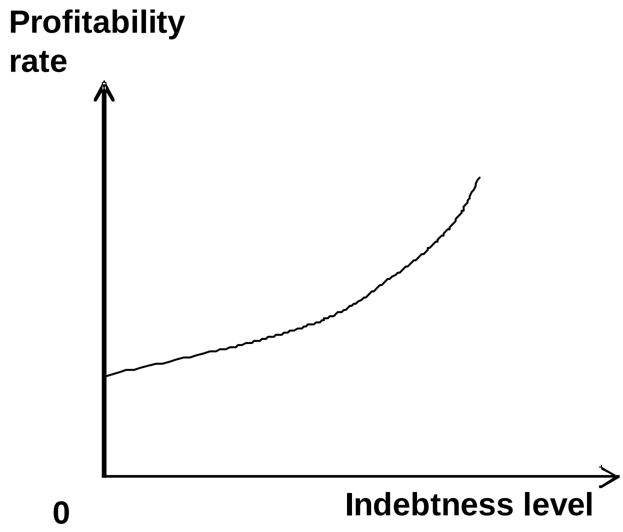

Exhibit 3: Operative EBIT rate higher than liability cost

This is the only case supposed by classical authors such as Ezra Solomon [8]. However, it is only one of the three possible cases and not the universal hardcore in a general theory on financing. These are the three possible cases pertaining the main quantitative relationship of business financing. Summing up, we obtain Hypothesis 2, as follows:

Hypothesis 3

The main quantitative criterion for financing decision-making is: profitability on net worth is higher whenever operative EBIT on assets is higher than liability cost.

Its demonstration is developed in *Appendix 3*.
Using internal accountancy values, this expression shows profitability rate in algebraic terms:

$$g = ebitV + (ebitV - k(i)) \cdot \frac{L}{NW} \qquad (10)$$

where:
$ebitV$: Operative gains (EBIT) per asset
V : assets Using some external market values, this expression may be put forward as follows:

$$g = ebitV \cdot (1 - k(i)) \cdot \frac{D_j}{A_j} \qquad (11)$$

What are the consequences of this formula? In the one hand, provided there is no indebtness, profitability is equal to operative EBIT on assets. We shall find the same consequence whenever liability cost is equal to operative EBIT on assets (as in Case (1)).

On the other hand, operative EBIT on assets should be higher than liability cost in order to achieve higher profitability on net worth (as in Case (3)), as follows:

$$[ebitV - k(i)] > 0 \quad iff \;\; ebitV > k(i) \qquad (12)$$

Finally, whenever operative EBIT on assets is lower than liability cost (as in Case (2)), then profitability is lower, or even negative, as follows:

$$[ebitV - k(i)] < 0 \quad iff \quad ebitV < k(i) \qquad (13)$$

From a general standpoint, HME was a first step to promote probabilistic studies on finance by means of econometric models. But the complexity of this matter suggests the use of hypothesis tanking into account not only probabilistic subjects, but deterministic ones and uncertainty as well, especially in relationship with the real economy. Useful probability distributions may change too, besides the traditional Gaussian one, and its variance may change now and then with material news, suggesting the use of heteroscedastic models. Multi-variable models, such as APT, may prove more comprehensive in order to reflect such a complexity than one-variable models, such as CAPM, in the field of financing decision making.

Such a model with coefficients aiming for financing evaluation may be put forward as follows:

$$\boxed{K_{it} = k_{0t} + \sum_{n=1}^{Z} [b_{in} \cdot f_{nt}] + \varepsilon_{it}} \qquad (14)$$

where:

K_{it}: cost of capital for enterprise i in period t

k_{0t}: expected cost of financing for a free-risk company

ε_{it} : value of pertaining idiosyncratic risk

b_{in}: sensitivity of instrument i to risk n

f_{nt}: value of incidence of risk factor n; i.e.: the difference between cost of systemic fact n minus risk-free rate.

Then, by means of a second regression, the formula for expected cost of a financing instrument is:

$$\boxed{EK_{i(t+1)} = \lambda_{ot} + \sum_{n=1}^{Z} (b_{in} \cdot \lambda_n)} \qquad (15)$$

where:

$EK_{i(t+1)}$: expected cost of enterprise i in lapse t, when: $i = 1,\dots,N$

b_{in}: sensitivity to risk n for instrument i, when: $i = 1,\dots,Z$

λ_n: risk premium for factor n. It is supposed to be the same for every instrument in a class. λ_{ot} is equivalent to the risk-free rate.

Business practice has brought increasing doubts about this scheme empirically. Additional ad-hoc hypotheses have been proposed to reflect certain cases.For instance, research sponsored by the U.S. Financial Executives Institute, a subsidiary of International Association of Financial Executives Institutes, said that the scheme of Modigliani and Miller presents a world too simple and does not cover important aspects such as agency costs, significant incentives, dividend policies or information asymmetries. By contrast, this research describes the cases of several U.S. corporations in successful or unsuccessful efforts to achieve a better balance of risks and financing costs [9].

But I have taken another approach: making contributions to develop another theoretical framework more comprehensive than the appointed, so as to achieve greater predictive efficacy.

Then, the discrepancy between the statement about the irrelevance of decisions on the financing structure and the practice of financial management is not due to tax effect on the payment of interest, but an argument in favor of considering the complexity of such decisions, which is not always trivial and which may involve a range of changing qualitative and quantitative factors.

Qualitative Factors

Financing decision frequently involve factors associated with the real economy, where markets are often non-efficient in many phases of their evolution and for different business

In such cases, there may be relevant market factors, regulatory issues, liquidity or debt concerns, and risks of various kinds, but the most characteristic factor is the political power given to whoever provides a significant portion of financing for an organization. That is often the reason to financing by debt as means of first resort.For instance, costs associated with bankruptcy risks [10]. They should limit excessive indebtness, but this is not the main variable for a general theory of financing because it depends on different and changing legislation for every country.

J. Stiglitz has stressed the possible importance of asymmetric information [11] Herbert Simon has classically stressed the need to obtain a satisfactory objective instead usually happens. Some authors have been stressing the importance of different internal or external of maximizing it because of organizational and social restrictions and uncertainty [12] Gordon Donaldson has put forward that a company's profitability and growth not always has the same consequences for shareholders, because of survival and power trade-off or pecking-order [13]. This problem may be associated with Agency Theory by Michael Jensen and William Meckling [14]
Merton H. Miller has followed this idea pointing the preference for indebtness instead of net worth, due to capital restrictions, possible voting power dilution and better management efficiency control [15]

Quantitative Factors

As I demonstrated algebraically in the book discussed, the Second Proposition (4) should be corrected as follows:

$$\boxed{l_j = \left[\theta_k + (\theta_k - r)\cdot\frac{D_j}{A_j}\right]\cdot(1-t)\ldots(6)}$$

Furthermore, this second proposition may be put forward considering other nomenclature that reflects the need to rely not only on exogenous market information but also on certain endogenous information like operating profits on assets whose perceived value hardly be arbitrated instantly. The formulation with the nomenclature based on exogenous and endogenous information is:

$$\boxed{l_j = \left[\frac{OP}{A_j} + \left(\frac{OP}{A_j} - r\right)\cdot\frac{D_j}{A_j}\right]\cdot(1-t)\ldots(7)}$$

where: *OP*: operating income is provided by the assets of a company that is perceived as endogenous information. This formulation is more comprehensive than MM Propositions, which can be considered a special case of it, since it allows considering the cases of all types of large or small, all kind of business, of more or less efficient markets, and lifting of the supposition that there is always a borrowing cost cheaper that profitability of corporate resources.
Given the complexity of financing decision, in order to evaluate probabilistic models I have proposed to supplant W.Sharpe´s CAPM in estimating the cost of a single financial instrument, for a new model based on the S.Ross´s APT. This allows considering that there are several factors involved (being multiattribute) and that its incidence varies over time (allowing heteroscedasticity conditions).

Rebuttal of the Importance of Tax Shield

The tax effect has a central role in MM Propositions. These authors have recognized that their conclusion about the irrelevance of the financing structure for determining the cost of capital in practice is belied by the behavior of financial managers. They attributed the reason for this discrepancy to a fundamental factor: the effect of income t

In this new book, I have demonstrated in a logical way and have illustrated through examples that Tax Shield is not valid as a general principle for a theory of financing.

In particular cases where there is a differential tax treatment that benefits some particular source of funding, the same should be considered for the respective decisions, but it is also the case for any differential cost of production, trade or another nature. The valid reason for the use of tax factor *(1-t)* is to standardize *k(e), k(i)* to the extent that the former usually is shown after taxes and the second before taxes, so that both costs are computed after taxes.

The subsequent secondary distribution of operating income to the IRS and creditors do not change the meaning of the financing decisions, although it might contradict some intuitive concept that in this case is incorrect.From a pragmatic point of view, Proposition III indicates that a firm would benefit from greater market value to the extent that achieves increased multiplication of the amount of its debt and its direct taxes, therefore subject to jurisdictions with higher rates of taxation. The predictive efficacy of this proposition is not empirically validated.

On the first multiplier, the amount of indebtedness, its elevation grows the level of financial risk, up to a tolerable limit for the market that the authors have designated as a level *L* empirically determined for each sector of the economy, without delving into its causal explanation.

However, companies with high debt levels have not higher ratings and higher valuation than less indebted firms, but the reverse.

On the second multiplier, the income tax rate, it is a notorious practice of companies that do not use resources to settle in areas of high taxation in order to increase its value, but quite the opposite. Therefore, this theoretical scheme inference is also refuted by empirical evidence.

An Example

Examples of the alleged Tax Shield have circulated repeatedly in research and extension work. One is found in an excellent book by James Van Horne [16], where it shows this case: the firm A has a debt of $ 1,000 to 2 % annual interest, the firm B is financed entirely with equity and the income tax rate on profits is 25%. Based on these data, he proposes the following figures:

CONCEPT	FIRM A	FIRM B
Sales	1000	1000
Cost of Sales	-800	-800
Operating Profits	*200*	*200*
Interests at 2%	-20	0
Profits before taxes	*180*	*200*
Taxes at 25%	-45	-50
Net Profits	*135*	*150*

Van Horne says that, as it is generally accepted, Company B obtains higher profits. It also states that this increased profit was $ 15, following the tax savings (i.e.: 20 * (1 - 0.25) = 15).

He adds that if you should sum the credit cost (or $ 20 only for case A) with shareholders profits (or $ 135 for case A and $ 150 for B), generating $ 155 in A versus only $ 150 only in B, attributed to tax savings of $ 5. Finally, using MM Proposition III says that if debt is permanent, the current value of that benefit when time tends to infinity is a resulting Tax Shield of $ 250.

In summary, the author attributes the difference in performance for the financing structure to the tax factor. However, in line with previous arguments, it will be shown that the difference actually originates in that the cost of borrowing (in this case 2%) is below operating profit on assets.

To assess this, firms must have some level of equity and borrowing to finance a given level of assets, in business reality. One could assume any numbers and this check would go the same lines; for instance, an asset of $ 2,000, which is financed either with a debt of $ 1,000 plus a net worth of another $ 1,000 for A, or entirely with equity of $ 2,000 in the case B. Any amount allegedly throws the same conclusion, undoubtedly, increasing the final return with a higher debt, since the cost of it is lower than operating profits on assets in this example.

However, the most that shareholders can earn is $ 150, operating income less income tax. The $ 50 of taxes can be distributed in different proportions between the creditors and the IRS, but are totally lost whatever financing structure employed. Shareholder return is to maximize, i.e. the percentage of final net income on invested capital; it does not make sense to add his income to that of creditors, as a variable to maximize. In this case, firm A obtains a 13.5% return on net assets (i.e.: 135/1.000) while firm B only gets a return of 7.5% (i.e.: 150/2.000). So the return of A is 80% higher than B´s (i.e.: [0.135 - 0.075] / 0.075 = 0.80). The proposed new formula (7) is adequate to provide for the profitability of firm A. obtaining a yield of 13.5%, name

Instead, MM formula (4) of the tax incidence would not infer profitability of shareholders, since it predicts a yield of 16% instead of 13.5%, namely $= 0{,}10 + (0{,}10 - 0{,}02) \cdot (1 - 0{,}25) = 0{,}16 \neq 0{,}135$

So the key issue is that the goal to maximize is return to shareholders, which is a percentage of invested capital and not an absolute scale. The same is enhanced because the operating income on assets is 10% (200/2.000 = 0.10) which is higher than liabilities costs (defined by 2%).

CONCEPT	FIRM A´	FIRM B´
Sales	1000	1000
Cost of Sales	-800	-800
Operating Profits	*200*	*200*
Interest at 2%	-20	0
Profit before Taxes	*180*	*200*
Taxes at 0%	0	0
Net Profits	*180*	*200*

The tax factor is irrelevant. To highlight this by a counterexample, let us assume removing income tax, meaning that its value is zero (0%). In that case, new figures can be seen in this Table. Now the profitability for firm A' would be 18% (i.e.: 180/1.000) while for firm B' would be only 10% (i.e.: 200/2.000) to coincide with its operating income on assets. Thus, profitability for firm A ' would continue to be 80% higher than for firm B' (i.e.: [0.18 - 0.10] / 0.10 = 0.80) although in this case the incidence of the income tax factor is null.

Consequently, the effect of the alleged tax shield is non-existent for the example in a given year, and therefore will also be absent for the infinite future years.

Conclusion

MMP put forward that capital structure should not have any consequences whatsoever to capital cost and therefore to a company market value, as per EMH.

However, those authors noted a priority by financial managers to resource to instruments representing indebtness in the first place instead of instruments representing net worth. Empirical test of an EMH consequence suffered from comparison with business practice.They found a third party as responsible. Governments push companies to maximize reasonable debt because of tax shield.

However, in this paper it has been demonstrated that it is not the case. Taxing concerns may be only a secondary reason to favor some financing structure depending on different taxing laws and deductions in different countries and phases of the business cycle.In fact, tax shield for an infinite term tends to tax rate times debt. However, market prices are not better for companies neither with higher in debtness nor with expensive tax rates, but the opposite factors.

Summing up, the explanation is based on the rather complex nature of the financing decisions. We may find quantitative as well as qualitative factors pertaining this subject. A material qualitative factor is the political influence associated with capital structure.

It should be noted that financial markets are not always efficient, such as in cases of emerging markets, small companies, market segmentation for promotional reasons, degrees of risk aversion, and in general during special phases of the business cycle where an coherent upward or downward tendency can be found.

As per this paper, the main quantitative factor in a general theory is that the expected operative earnings relative to assets should be higher to liabilities cost in order to enhance profitability on net worth. The main qualitative factors ought to refer to financial risks and political concerns pertaining financing decisions.

The complexities of financing decision making might be studied by means of heterocedastic and multivariate probabilistic models, including some deterministic and uncertainty factors as well. But so-called tax shield should not be taken into account as the hardcore variable that enables reasonable explanations of financing decisions. I was surprised by these new theoretical contributions and am looking forward to receiving comments from colleagues.

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Appendix A: Demonstration of Hypothesis 1

Hypothesis

In a general theory of financing, the incidence of indebtness cost in cost of capital is not determined by the income tax factor, since income taxes influence the whole cost of capital.

Demonstration

Definition of net gains begins with operative gains. In first place, interests and similar charges concerning indebtness are deducted from them. In second place, gains taxes are deducted from the former net amount. In algebraic form, it can be shown as follows:

$$E = (EBIT - I).(1-t) \qquad \textbf{(A.1)}$$

where:

E : Net earnings, after interests and taxes.

$EBIT$: Net operative gains obtained from a company's assets resources.

I : Interests, including every indebtness cost under various names.

t : **T**ax rate on earnings, which charges the whole taxable gain, i.e. the difference between operative gains minus financing charges

Above mentioned concepts are expressed by variables of absolute values. If divided by the variables pertaining their concepts of origin, the relative values are obtained, as follows:

$\frac{E}{NW}$: Net earnings on net worth *(NW)*, also known as rate of profitability, which is the *ex post facto* main variable to be maximized on the long run and in a feasible way for a free enterprise system.

$\frac{I}{L}$: Interests on liabilities *(L)*, also known as interest rate or indebtness cost, which Modigliani and Miller name *r*.

$\frac{EBIT}{V}$: Operative earnings on assets (or: *ebitV*). Operative earnings are defined before deducting liabilities costs and gain taxes, as well as other non-operative expenses.

We shall show the correct formula of tax factor on indebtness cost and net worth yield based on these relative concepts.

At first, we divide both members of equation (A.1) by net worth (PN) and obtain this expression.

$$\frac{E}{NW} = \frac{(EBIT - I)\cdot(1-t)}{NW} \qquad \textbf{(A.2)}$$

Using distributive property, we have:

$$\frac{E}{NW} = \left(\frac{EBIT}{NW} - \frac{I}{NW}\right)\cdot(1-t) \qquad \text{(A.3)}$$

We can reformulate two components of the second term of equation (A.3) and obtain equivalent expressions representing important concepts:

$$\frac{I}{NW} = \frac{I}{L}\cdot\frac{L}{NW} \qquad \text{(A.4)}$$

$$\frac{EBIT}{NW}=\frac{EBIT}{(L+NW)}\cdot\frac{(L+NW)}{NW}=\frac{EBIT}{(L+NW)}\cdot\left(1+\frac{L}{NW}\right) \qquad \textbf{(A.5)}$$

Replacing the above mentioned variables by these new expressions in formula (A.3):

$$\frac{E}{NW}=\left[\frac{EBIT}{(L+NW)}\cdot\left(1+\frac{L}{NW}\right)-\frac{I}{L}\cdot\frac{L}{NW}\right]\cdot(1-t) \qquad \textbf{(A.6)}$$

Using distribution:

$$\frac{E}{NW}=\left[\frac{EBIT}{(L+NW)}+\frac{EBIT}{(L+NW)}\cdot\frac{L}{NW}-\frac{I}{L}\cdot\frac{L}{NW}\right]\cdot(1-t) \qquad \text{(A.7)}$$

Then, we can aggregate using common factor $\frac{L}{NW}$, as follows:

$$\frac{E}{NW}=\left[\frac{EBIT}{(L+NW)}+\left\{\frac{EBIT}{(L+NW)}-\frac{I}{L}\right\}\cdot\frac{L}{NW}\right]\cdot(1-t) \qquad \text{(A.8)}$$

We have arrived to equation (10) of main text.

Appendix B: Demonstration of Hypothesis 2

Hypothesis 2

A given relationship between interests and cost of capital is equivalent to that very relationship between interests and cost of net worth

Demonstration

For every $k_{(0)}, k_{(i)}, k_{(e)} \geq 0$:

$$k_{(i)} \leq \mathrm{k}_{(0)} \qquad \text{(B.1)}$$

$$k_{(i)} \leq \frac{k_{(e)}\cdot PN + k_{(i)}\cdot P}{PN+P}$$

$$k_{(i)}\cdot PN + k_{(i)}\cdot P \leq k_{(e)}\cdot PN + k_{(i)}\cdot P$$

$$k_{(i)}\cdot PN \leq k_{(e)}\cdot PN$$

$$k_{(i)} \leq \mathrm{k}_{(e)} \qquad \textbf{(B.2)}$$

The same demonstration in inverse order shows that ($k_{(i)} \leq \mathrm{k}_{(e)}$) implies ($k_{(i)} \leq \mathrm{k}_{(0)}$) with the same relationship between variables.

The same argument is valid when interest is bigger than or equal to cost of indebtness, i.e. $k_{(i)} \geq \mathrm{k}_{(0)}$ implies $k_{(i)} \geq \mathrm{k}_{(e)}$. In inverse order: $k_{(i)} \geq \mathrm{k}_{(e)}$ implies $k_{(i)} \geq \mathrm{k}_{(0)}$.

Therefore, for $k_{(0)}, k_{(i)}, k_{(e)} \geq 0$:

$$(k_{(i)} \leq \mathrm{k}_{(e)}) \Leftrightarrow (k_{(i)} \leq \mathrm{k}_{(0)}) \qquad \text{(B.3)}$$

as well as:

$$(k_{(i)} \geq \mathrm{k}_{(e)}) \Leftrightarrow (k_{(i)} \geq \mathrm{k}_{(0)}) \qquad \text{(B.4)}$$

Appendix C: Demonstration of Hypothesis 3

Hypothesis 3

The main quantitative criterion for financing decision-making is: profitability on net worth is higher whenever operative EBIT on asset is higher than liability cost.

Demonstration

Profitability rate is defined by the quotient of net earnings on Net Worth. Net earnings may be shown as the difference of operative gains minus debt cost, as follows:

$$g = \frac{E}{NW} = \frac{EBIT - k(i) \cdot L}{NW} \qquad \text{(C.1)}$$

where:
g : Profitability rate
E : Final net profits
NW : Net worth
$EBIT$: Operative earnings before interests and income taxes
$k(i)$: Liability cost rate (including every associated cost)
L : Liabilities
Assets are equal to liabilities plus net worth. Since operative EBIT was defined as follows:

$$EBIT = ebitV \cdot V \qquad \textbf{(C.2)}$$

where:
$ebitV$: operative EBIT divided into assets
V : value of assets

Then, this substitution may be put forward:

$$g = \frac{ebitV\left(NW + L\right) - k(i) \cdot L}{NW} = \frac{ebitV \cdot NW + ebitV \cdot L - k(i) \cdot L}{NW} \qquad \text{(C.3)}$$

Profitability rate is then obtained by simplifying:

$$g = ebitV + \left(ebitV - k(i)\right) \cdot \frac{L}{NW} \qquad \textbf{(C.4)}$$

We have arrived to equation (14) of main text.