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REVIEW ARTICLE

Mathematics in Economics: Necessity or Sufficiency?

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Abstract

This paper pursues an epistemological analysis of the synergy between Mathematics and economics and its evolution over time. Throughout the study I will develop a succession of ideas, in order to emphasize the importance of Mathematics in Economics, also having the objective of answering the question "Is it necessary to apply Mathematics in Economics? The study is based on a series of articles and studies of those who have influenced the science of economics, as well as critical opinions of scientists that have interacted either directly or indirectly with the theories of the above mentioned authors. The indisputable role that Mathematics has in economics should be accepted and taken into account by all the actors of the contemporary society, regardless of the individual's area of expertise, matter that would however fulfil the guidelines for any discipline, given that the remarkable progresses in fields of natural sciences from the Renascence to the present time are mostly due to the fact that all the laws of nature can be expressed as mathematical equations, and handling this equations represents the foundation of modern science.

Keywords: *Economics, Epistemology, Mathematics.*

Introduction

Although known from the most ancient times of human society [1], interdisciplinary as phenomenon has only been defined and emphasized along with its spread and rapid evolution registered during the second half of XX century [2]. Therefore, beside the technological development and the increase of demarcation between the economical – social areas, an increasing need for optimization appeared, with a solution in combining certain methods and techniques specific to several sciences, and applying them within their interest areas. Such phenomena have been encountered in Economics when, in order to explain rules such as the theory of the general balance or the theory of marginal utility, mathematic methods based on economical thinking were used. This way it was proved that, in practice, Mathematics are not a substitute, but a complement for the economic sciences. This paper provides an epistemological analysis in terms of the synergy between Mathematics and economics and its evolution over time. More exactly, throughout the study I will develop a series of ideas, in order to emphasize the importance of Mathematics in Economics which will also serve the objective of answering the question "Is it necessary to apply Mathematics in Economics?" In order to accomplish the set objectives, in the first part of the paper I will review some of the main authors that have marked out the economic science having based their papers on Mathematics, followed by the epistemological position of Mathematics in Economics in the second part. The third part will be a discussion over the ideas that will have been developed in the paper; the article ending with a sum of conclusions, recommendations, and future lines of research. The methodology used for achieving these goals is qualitative, as the study keeps track of the evolution in time of the economic science influenced by the mathematic science. The study uses articles and works of those that have undeniably marked the economic science as well as critical opinions of those that have interfered either directly or indirectly with the theories of the above mentioned authors.

Mathematics in Economy

Understanding the nature and role that mathematic Economics has is not the same with understanding the connections between Mathematics and Economics. The presentation of the mathematic Economics involves both essentialist arguments related with the true nature of economic objects and the Economics as a whole and the presentation of arguments aimed to emphasize the suitability of Mathematics within the domain of Economics, since the last one is of

quantitative origin. Mathematic methods and techniques are not secluded by the social sciences, nevertheless by Economics, these being used since the seventeenth century, usage that due to its increased frequency has led to the appearance of Statistics as a science. Moreover, the term of statistics itself, firstly created to define the quantitative method for describing Economics by designing the assembly of instruments and techniques specific to Mathematics, used for collecting and interpreting the information, has boosted this branch derived from Mathematics to the rank of science, from which Econometrics later broke off. Amongst the economists that have used Mathematics in their activity of economic research, we can mention famous names such as Augustin Cournot, Leon Walras, Vilfredo Pareto, Paul Samuelson, Francis Edgeworth. From the mathematicians that have decisively influenced the evolution of Economics we could enumerate Karl Pearson, William Gosset, known under the pseudonym of „Student", Udny Yule or Nicolae Georgescu Roegen. Either the above were mathematicians but they have significantly influenced the science of Economics, either vice-versa, their work has served as foundation for the subsequent economic theories and, implicitly, for the modern Economics in its current stage. An important contribution brought to Economics but based on mathematic techniques is the one come from the French economist Leon Walras, considered as the greatest economist of all times [3] due to his contribution on the theory of marginal utility and the general equilibrium theory. Son of an economist and professor, having spent his youth in Paris as writer and art critic, Walras began pretty soon to follow the footsteps of his father from several points of view, including applying Mathematics in Economics. His most important paper, „Elements of Pure Economics" is printed out in 1874, while he was at the Academy of Lausanne, three years after Jevens and Manger had published their papers. Along with publishing his ideas, Walras tried to build a network of correspondence with almost every great economist of the time, in order to promote his newly developed theory, being mostly ignored by the contemporary economists and mathematicians. Although succeeded by another two volumes – "Studies on Social Economics" and "Studies of Applied Economics", volumes considered by the author as complementary to the initial one, Walras will always be remembered by the economists with his first paper, the rest being considered either too casual in terms of information, either not correlated with the main piece of work, which was impeding the true walrasian vision from being applied [4]Antoine Augustin Cournot, philosopher and mathematician of the XIX century, through the view of published theories, is considered as one of the pioneers of the domain of the economic demand and supply [5]. Furthermore, Cournot is the one that introduces the concepts of elasticity and marginal cost, in his work "Researches on the Mathematical Principles of the Theory of Wealth", concepts that have significantly influenced Walras.Vilfredo Pareto, Italian economist and philosopher, disciple and collaborator of economist Walras, joined those that have based their economic theories on Mathematics, using techniques and mathematic instruments for each of his important discoveries [6]. Born in a family of exiled nobles to Paris, having graduated in Toronto, Pareto comes to the University of Lausanne, where he will be spending the rest of his life and where, in 1893, he will become the successor of his mentor Leon Walras, whom he replaces for the management of the Economics department. His significant theories will be published starting with his admittance at the university, enclosed in volumes such as "Cours d'économie politique" (1896) or "Manual of Politic Economy" (1906)Paul Samuelson, economist coming from a well known family in the field, who described himself as "the last generalist economist, with interests that range from mathematical Economics down to current financial journalism", has been the second laureate of a Nobel award for Economics in 1970 and one of the founders of neo – keynesism. Through the contributions he brought to economic science, some of them based – as himself stated – on the classic methods of Gibbs' thermodynamic, he managed to be surnamed the father of modern Economics [7] or the greates academic economist of the XX century [8], writing about one paper per month, most of them technical, for o period longer than 50 years, including his best known of his works „Economics: An Introductory Analysis" which became the best sold economic book of all times.There is no doubt that Economics would not have had the aspect we all know without explaining the consumer's behaviour by means of indifference curves, whose theorization was made by Francis Ysidro Edgeworth at the end of the 19th century, beginning of 20th century. Amongst his contributions, the Edgeworth Box and the volume „Mathematical Psychics: An essay on the application of Mathematics to the moral sciences, 1881" represents an irrefutable evidence of the craftsmanship he has proved when applying the mathematic methods in Economics. Some mathematic epistemologists have claimed that certain representatives of social sciences make use of Mathematics only to make their discipline appear as science. To fight this idea, a brief review of the names of Nobel laureates would be

sufficient having in sight that the most of distinctions have been awarded for mathematical models applied within the area of Economics. As example, there is a component named "the continuous time" [9] in the field of finances[1], which is derived from Mathematics with the purpose of studying the stochastic phenomena that can be included in mathematical models. Furthermore, during the last years, a new discipline has been developed, named Financial Engineering, also functioning based on specifically mathematic techniques and instruments, having been implemented by a team of both mathematicians and economists, thing that pointed out the already obvious connection between Mathematics and Economics. This area of Economics, with an interest in financial markets, epistemologically different from other disciplines within finances by the interest shown to the stochastic processes behind the economical phenomena and not by the value of certain indicatives, is based on the Brownian movement and the writings dating from the end of the nineteenth century. The connection between Mathematics and Economics is centuries old. During the twentieth century, especially in the first half, the mathematical language became the main form of expression for economic phenomena, fact also showed by the impressive number of economist's laureates with a Nobel award due to mathematical papers. From an epistemological perspective, just as the science of Economics has evolved into a system of economic sciences, the disciplines with which Economics interacts have specialized in certain domains, this way resulting Economic Mathematics, Cybernetics, Classical Econometrics or Modern Econometrics, Informatics in Economics, Financial Mathematics, Actuarial Mathematics, Operational Research, Mathematical Statistics, etc. On what concerns the hybrids of Mathematics, these are advancing and developing in such an fast rhythm that, epistemologically, the bounds are slightly delimited, causing the answer to the question "Is Financial Mathematics the same thing with Mathematic Finances?" to continuously modify. Regardless of the answer to this question, all the disciplines are tangential with Mathematics and Economics, having this way a dual character resulted from the dogma of consecrated sciences. Another point of view from which we can analyze the impact of Mathematics in Economics is that of catalyst for other sciences. This way, Mathematics represents a link between the natural and/or social sciences connecting Physics or Chemistry with Economics, starting with the number of Avogadro to the Brownian Model, creating the premises for the appearance of other economical disciplines, based on mathematical techniques but with concepts of chemical or physical origins. Moreover, the relation between Mathematics and Economics can be described through the separate intellectual activities taking place in the two domains which, from epistemological point of view, can be accomplished by analyzing and interpreting the ideas and practices from each science according to the premises and the laws by which they are governed. The history of Mathematics presumes updating the mathematical knowledge as new theories are developed, new research areas lead off, and new techniques are implemented. In addition, history also implies a metamorphosis of the vision over the mathematical knowledge; for example, the change of perspectives over the nature of mathematical objects, of what evidence represents or how the rigor is defined, what is necessary versus what is unnecessary, etc. Analogically, the history of Economics comprises not only the epistemology of Economics, but also the changes occurred in time: what Economics represents, what is considered to be a relevant explanation in Economics, what does empiric work represent, or what defines a viable economic model. From a different point of view, a discussion over the implications that Mathematics has in Economics must reflect not only the changing comprehension about the content of mathematic knowledge but also a change of the outlooks beliefs that economists have regarding the image of this knowledge. The distinction between the content and the image of knowledge offers a different perspective over the relation between Mathematics and Economics. For this reasoning to be accessible to any economist, it must be taken into account that three distinct transformations of the image of Mathematics have occurred from the beginning of the XIX century until the end of the XX century [10].As a starting point, the conditions and perspectives that Mathematics had at its appearance at the beginning of XIX century will be taken into account. At a closer look, especially in England, it can be noticed the indisputable importance of the "Elements" of Euclid and the "Principia" of Newton, which writings are considered to have set up the stage for Mathematics. From Euclid it was clear that Geometry represents the paradigm of Mathematics, also representing a path to the truth. Learning Geometry represented a modality of understanding how rigorous arguments can lead to the truth and the one studying Mathematics in general – Geometry in particular

[1]The study of financial assets and such derivates begun with Louis Jean-Baptiste Alphonse Bachelier (1870–1946), who was a French mathematician. He is now generally considered the first to have developed the model of Brownian movement in his doctoral dissertation, even before Einstein

was getting into deducing the truth about the surrounding world, raising this way Mathematics to the rank of paradigm of deductive thinking and logical reasoning. Similar to this point of view according to which deductive reasoning based on real premises could lead to real conclusions, Newton suggested how this type of Mathematics could also help with the understanding of the physical world. This vision over Mathematics is the root of mathematical models of Ricardo, also present in the works of Whewell about the Economics based on Mathematics. The Economics of those times was using a specific type of Mathematics, Euclidean Geometry, in order to demonstrate his theories. Marshall – successor of the above mentioned authors - has acted in the same manner as Newton, by using geometrical arguments for his statements. His theories instead, being stated in the second half of the 19th century, have required from their developer knowledge of both Euclid and Newton work**.** The first change of the image of Mathematics appeared as an effect of a new concept regarding the Mathematic truth in the second third of the 19th century, based on the notions of Whewell, according to whom Mathematics were a paradigm of some sure knowledge. The amplitude of the non-Euclidian geometries has caused Whewell's arguments about axioms and implicitly about truth to pass as ungrounded much before passing in the 20th century. During the new geometries, the difficulty of connecting the mathematic truth with a certain (Euclidian) geometry has proved to be a real confidence test for the educational Victorian practices [11], that had to associate Physics arguments to each Mathematic reflection, image otherwise representative for the economists of the time, including Edgeworth or Pareto. Later on, along with the development and diversification of the mathematic disciplines due to the increasing number of alternatives for Euclidian geometry and the recognition that only a set of axioms could not be enough to state the mathematic truth, a new paradigm has been developed, according to which, in order for a mathematic model to be considered correct and valid, it must receive concrete physical interpretations. In other words, most of the 19th century Mathematics has been based on geometric evidences founded on assumptions, named axioms, which could have been linked to certain optimization processes associated to concrete physical systems. Thereby, it was only in the last decades of the 19th century that the real mathematic models based on economic behaviour were encountered. An edifying example in the field of Economics is given by the marginal revolution, model based exactly on this new type of reasoning. At the beginning of the 20th century, the mathematic perception started changing once again in response to the new challenges from the field, process that has obviously reverberated over Economics. Thus, within the mathematic dogma, certain discrepancies in the foundation of logic and arithmetic have appeared, causing the image of Mathematics to seize a new form at the very moment when economists were beginning to understand that for each economic theory it was required a concrete argumentation, mechanical reasoning and a quantification of sizes in order to concatenate the economic behaviour, transformation that has implicitly spread in the field of Economics too. This way it was attained that the mathematic truth would be reached not according to the physical reasoning but according to some other mathematic theories, which for economist implied during the inter-war period the need to reflect over the previous perspectives related to the matter of economic modelling from the end of the 19th century, following that, later on, the requirements of the Second World War regarding the researchers, the engineers and other social scientists would move back from axiomatization to what was to be called "operational research" on hypothetical, perfect economic agents. After the Second World War and during the Cold War, as new areas appeared and extended, the conceptions of the economists related to the usage of Mathematics were about to change again, causing Economics and Microeconomics, as well as the techniques and knowledge applied from Mathematics to become the new rebuilt core of economic sciences. At the International Congress of Mathematics from Berlin, August 1998, the issues of pure versus theoretic Mathematics was raised, each of them having a large number of followers. Thereby, within Economics, it is considered that a model, an accurate and relevant research is one that turns to be valid in both theory and practice. An important usage of Mathematics in Economics is the one related to the interpretation of the decisions made by consumers, explaining this way the consumer's behaviour, based on practical applications. Furthermore, the whole economic chain related to the maximization of one company's profit is influenced by Mathematics and the production itself could be described as a conversion of some variables (inputs) in others (outputs). This way, the quantitative relations resulted from this interaction are determined by a set of alternative available technologies. As new algorithms were developed, the differential calculus and elementary algebra – the two traditional instruments of economic mathematic - are replaced or at least supplemented by those of topology and Matrix Algebra. Explaining the consumer's behaviour is made in parallel from

several points of view. The households are regarded as companies, with an income derived from selling the personal services or rights and an afferent consumption. The restraint imposed by the household's budget will drive them to choose that combination of products and services that would bring them the highest level of satisfaction. If the previous theories were treating utility as a quantifying parameter, at a more thorough analysis, measurement of cardinal utility has proved dispensable for composing the problem of Maxim on one hand, and not veracious when transposing it into practice on the other hand. Thereby, the intervention of Mathematics was more than desirable, being able to crystallize the analysis around the axiomatic formulation of the properties of social welfare described as the level of utility reached by each member of the society. Another angle to analyze the epistemological impact of Mathematics on Economics could be that of the contribution it brings to a certain branch of Economics – Marketing. Within this sector, Mathematics is offering its assistance for a better understanding of the consumers' behaviour, through the predictions based on their profiles. Also, based on mathematic algorithms as well, the large companies can calculate the expected level of sales, depending on the market consumers' preferences or the percentage that they should invest in advertising in order to raise the sales level. To emphasize the importance and impact that Mathematics had and continue to have for Economics, we can only conclude by saying "there has never been a better moment for being a mathematician[2]".

Conclusions

The need for Mathematics in Economics is more that obvious. Moreover, the creation of Mathematics as one of the primary sciences denotes its importance and the necessity of humanity to describe the world by means of figures. From this point of view, the spread that Mathematics has known in time can only be regarded as recognition of the dependence on Mathematics of all the surrounding things. Regarded either as method applied in different branches of science or as primary science, the role that Mathematics has for the 20th century society is indisputable, and the connection with the other sciences is more obvious than ever. Economics cannot exempt the above mentioned as it has been successfully based on the methods and principles of Mathematics, making it a catalyst and a powerful ally for any action taken. Quite probably, if the two sciences would have not joined composing elements, regardless of their nature, none of them would have met the current stage of development. Each new notion, each idea introduced by any of the two has been used as starting point for a research conducted by both Economics and Mathematics, obtaining this way an exponential domino effect that has finally propelled both Mathematics and Economics amongst the noble sciences. Furthermore, epistemologically speaking, when it comes down to how much Mathematics should be Economics applied or when it should be applied, I consider that, in fact, Mathematics and economics have formed a symbiosis, each one acting as "Alma Mater" for the other, offering new sources for theories, postulates, axioms, theorizations and explanations. Furthermore, I consider that any new theorization, any new concept or axiomatization in Economics, passed through the filter of mathematic, has proved a higher level credibility and its impact in society has significantly increased. Therefore, reverting to the statement according to which some disciplines are using Mathematics to call themselves science, without commenting it on general level, I would criticize this information through the angle of the interaction between Mathematic and Economics. This way, Economics, an absolutely indispensable science in the 21st century - especially since the resources are more and more precious and the ingrate equation limited resources unlimited necessities imposes a growing pressure on society – does not require or uses Mathematics to promote its image but cooperates with the last one to optimize the results. The indisputable role that Mathematics has in economics should be accepted and taken into account by all the actors of the contemporary society, regardless of the individual's area of expertise, matter that would however fulfil the guidelines for any discipline, given that the remarkable progresses in fields of natural sciences from the Renascence to the present time are mostly due to the fact that all the laws of nature can be expressed as mathematical equations, and handling this equations represents the foundation of modern science. Thereby, 150 years after their enouncements, the words of Walras can be quoted without the risk of considering them obsolete: "as for those economists who do not know any Mathematics, who do not even know what is meant by Mathematics and yet have taken the stand that Mathematics cannot possibly serve to elucidate economic principles, let them go their way repeating that 'human liberty will never allow itself to be cast into equations' or that 'Mathematics ignores frictions which are everything in social science' and other equally forceful and flowery phrases", making

[2] James R. Schatz, director of Mathematical researches, National Security Agency, USA

Mathematics a good candidate for la *reine des sciences*[3][12-16].

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