



RESEARCH ARTICLE

The Phillips Curve – The Case of Indian Data

Amaresh Das*, Frank Martin

*College of Business, Southern University, New Orleans, LA 70126, USA.**Corresponding Author: Email: adas2@cox.net

Abstract

The paper uses three different estimates of the output gaps to obtain estimates of the Phillips curve on the basis of the Indian data. We use a modified Hodrick-Prescott filter with a non constant smoothing parameter. We allow the smoothing parameter λ to vary over time so that its value reflects the nature and magnitude of supply and demand shocks in India, although determining the exact value still remains a problem.

Keywords: *Box-Cox method, Hodrick- Prescott filter, Phillips Curve, Potential output.*

Introduction

- In capitalistic economics, including the economy of India, aggregate economic variables experience repeated fluctuations about their long-term growth paths.
- One of the maintained hypotheses in macroeconomics, based upon growth theory considerations, is that the growth component of aggregate economic time series varies smoothly over time.

Growth in such economies is characterized by roughly proportional growth in (per capita) output, investment, consumption, capital stock and productivity (output per hour) and little changes in the hours of employment per capita per capita or house hold. The cyclical variations in output arise principally as the result of changes in cyclical hours of employment and not as the result of changes in cyclical productivity or capital stocks.

It is natural to think of some ‘potential output’ as the level of output that is consistent with the maximum sustainable level of employment: That is, it is the level of output at which demand and supply in the aggregate economy are balanced so that, all else being equal, inflation tends to gravitate to its long-run expected value. In view of the recent accelerating price rise in India, the question arises as to why stabilizing inflation does not lead to stabilizing output. The combination of the dual mandate and this definition suggests two reasons that estimating the path of potential

output is so central to the conduct of monetary policy.

To derive the answer we need to know the empirical relationship between inflation and output. First, to evaluate whether our policies will help achieve the maximum sustainable employment objective we need know the level of future output that would be consistent with that objective. Second, the level of output relative to potential output, which is referred to as the *output gap*, plays an important role in the inflation process. When the actual level of output is above potential output--so that the output gap is positive--labor and product markets are excessively tight; then, if things such as expected inflation and temporary supply factors are held constant, inflation will tend to rise. Conversely, when the output gap is negative and labor and product markets are slack, inflation will tend to fall. Estimates of the future path of potential output are therefore needed to assess whether the projected path of output that is implied by current monetary policy will lead inflation to move in a direction that is consistent with price stability¹. This is what is exactly the paper is intended to do...

¹ One caution is in order. Under present conditions potential output cannot serve, as it once did, as target for short-term economic policy with respect to demand managements. Its attainment would probably be accompanied by unacceptably strong upward pressure on prices, rather it is introduced here as an analytical tool. Moreover, the timing of the fluctuations in output per unit of input is different from that in total output. This is because employers cannot fully foresee changes in demand and do not adjust employment and working hours instantaneously when the course of demand changes. An effective way to isolate the effects of demand fluctuations is to construct series for output and determinants of output per unit of input as they appear under actual conditions but also as they would appear under ‘standardized’ or ‘potential’ demand conditions.

In measuring potential output, we can use the "natural rate" version of the Phillips curve, which followed from the seminal research of Nobel prizewinners Milton Friedman [1] and Edmund Phelps [2]. Friedman and Phelps demonstrated that there should not be a long-run tradeoff between inflation and unemployment and that the economy will gravitate to some natural rate of unemployment in the long run no matter what the rate of inflation is. In other words, the long-run Phillips curve is vertical, and attempts to lower unemployment below the natural rate will only result in higher inflation.² There is a limited or no research directly dealing with the existence of the Phillips Curve in India. Due and Gaur [3], Paul [4] Lipsa Ray [5] to name a few, used different methodologies to explain the over-all inflation dynamics in India, Some blamed it on the countercyclical monetary policy, some blamed it on the supply shocks. Lipsa Ray who uses the Hodrick-Prescott filter, a mathematical tool used in macroeconomics especially in real business cycle theory to separate the cyclical component of a time series from raw data, did not find any convincing evidence regarding the existence of the output gap

Given a successful filtering of the data, we can be represented with a metric to gauge the magnitude of the deviation of actual output from trend output. The Hodrick-Prescott filter is generally used to obtain as with the split time-trend method a smoothed-curve representation of a time series, one that is more sensitive to long-term than to short-term fluctuations. The filter was first applied by economists Robert J. Hodrick and Edward Prescott [6]. But, the filter has a misleading predictive outcome when used dynamically since the algorithm changes (during iteration for minimization) the past state (unlike a moving average) of the time series to adjust for the current state regardless of the size of λ used.³ We modified the filter in our study to adjust for the sensitivity of the trend to short-term fluctuations and this is achieved by modifying the multiplier λ .

The plan of the paper will be as follows: In section 2 we will compute the modified HP filter for

²According to the natural-rate hypothesis, there is a natural rate of unemployment--also more commonly referred to as the NAIRU (non-accelerating inflation rate of unemployment)--at which inflation tends to gravitate to its long-run expected value. A natural rate of output--that is, potential output--corresponds to the NAIRU.

³The conventional approach is to estimate potential output using a production function relationship and estimates of the factor inputs available to the economy. This requires more information and assumption about economic interrelationships but is less mechanical and more directly relevant to macroeconomic assessments.

Indian GDP and then estimate what is also known as the expectations – augmented Phillips curve with an aim to evaluate the explanatory powers of the output gaps. Section 3 will submit and interpret the results. Section 4 concludes the paper with some additional remarks

The HP Filter and the Theory

The fundamental time series models for trend analysis is one made up of a *stochastic trend* component,

$$y_t = \mu_t + \varepsilon_t \quad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2), \quad t \dots T \quad (1)$$

with

$$\mu_t = \mu_{t-1} + \beta_{t-1}$$

$$\beta_t = \beta_{t-1} + \varsigma_t \quad \varsigma_t \sim NID(1, \sigma_\varsigma^2) \quad (2)$$

The irregular and slope disturbances, ε_t , ς_t , respectively are mutually independent and the notation $NID(0, \sigma_\varepsilon^2)$ denotes normally⁴ and independently distributed with mean zero and variance σ^2 .

The signal noise ratio,

$$q = \frac{\sigma_\varsigma^2}{\sigma_\varepsilon^2} \quad (3)$$

plays the key role in determining how observations should be weighted for prediction and signal extraction. The higher the q the more past observations is discounted in forecasting the future. Similarly, a higher q means that the closet observations receive a bigger weight when signal extraction is carried out. [The trend is known as an *intergraded random walk* trend]. The HP filter is equivalent to the smoothed trend obtained from model (1) and (2). The filter has been applied to actual data, and in studies in which artificial data from a model are compared with the actual data [7,8]

One can consider the benchmark continuous-time version of equation (1) that satisfies the conditions stated that is, where the cycle as well as the second difference of the trend are independently and moderately distributed, taking

⁴The state-space Kalman filter approach also assumes normally distributed innovation to trend growth, see Harvey (1989). The HP-filter uses observations at time $t + 1$, $t > 0$) to construct the current time point t , while the recursive setting assumes that only current and past states influence the current observation. One way around this is to use the one-sided HP-filter, see Stock (1999).

the form of Brownian motion increments. We then analyze the change in variance when observing the process at discrete time intervals.

Let y_t be the 'flow' $\delta z_t = \tau_t \delta r + \sigma_t \delta w_t^1$ where

$$\delta \tau_t = \mu_t \delta t, \quad \delta \mu_t = \sigma_t \delta w_t^2$$

where δw_t^1 and δw_t^2 are two independent Brownian motions. There are two possibilities for observing the process at some time interval α ; these observations may be time aggregated (or time averaged) or they may be sampled at discrete time interval.⁵

The HP filter will decompose a time series (e.g., GDP) into growth and cyclical components

$y_t = y_t^g + y_t^c$ for $t = 1, \dots, T$ where y_t is the natural log of GDP and y_t^g and y_t^c are the growth and cyclical components, respectively.

- The measure of the 'smoothness' of the $\{y_t^g\}$ series the sum of squares of its second difference
- The series $\{y_t^c\}$ represents a deviation from the $\{y_t^g\}$ but under the framework considered, the long-run average of this deviation is zero
- The actual filtering technique is therefore represented as an optimization problem.

The decomposition assumes that series being de trended does not contain any seasonality and because the cycle is derived residually, it does not separate out the cycle from any irregular movement. The resulting cycle is measured with error. The HP filter minimizes the variance of y_t^c subject to penalty for variations in the

⁵ Consider observing for some length $\alpha > 0$

$$\int_{s=0}^{\alpha} \delta z_{t-s} = \tau_{t;\alpha} + c_{t;\alpha}$$

where

$$\tau_{t;\alpha} = \int_{s=0}^{\alpha} \mu_{t-s} \delta s$$

$$c_{t;\alpha} = \int_{s=0}^{\alpha} \sigma_c \delta w^1$$

For any stochastic process x_t define a α - differencing operator

$$\Delta_{\alpha} x_t = x_t - x_{t-\alpha}$$

We are interested in $\lambda_{\alpha} = \frac{\sigma^2(c_{t;\alpha})}{\sigma^2(\Delta_{\alpha}^2 \tau_t)}$

Changes with σ^2 One can equally divide the process by α and obtain time averaging rather than time aggregation.

second differences of the growth term. The filter is then given by

$$\text{Min} = \sum_t^T (y_t - y_t^g)^2 + \lambda \sum_{t=1}^T [(y_{t+1}^g - y_t^g) - y_t^g - y_{t-1}^g]^2 \quad (4)$$

The parameter λ controls the smoothness of y_t^g . The minimization of (3) provides a mapping from y_t to y_t^g with y_t^c determined residually. The estimates of potential output using the HP filter depend on the choice of λ , the smoothing parameter. A λ of zero corresponds to an extreme real business cycle model where all of the fluctuations in real output are caused by technology shocks because the HP trend would be the same as the series being de trended. Conversely, as λ tends to infinity the HP trend toward a deterministic time trend. Following Hodrick and Prescott we will set λ to 1600 for use with quarterly data but test the robustness with different values.⁶ Even if the HP filter is optimal for equation (1), it is unlikely to be optimal for equation (1), because time aggregate use introduces moving average terms. As our focus is on adjusting λ when changing λ the frequency of observations, we shall however, ignore the issue of optimal filtering; instead simply focus on the question of how the ratio of variances changes.⁷

Methodology and the Modified HP filter

This section modifies a la Razzak and Dennis [9] the HP filter by allowing the smoothing parameter λ to vary with time to reflect changes in the variance of aggregate demand and supply shocks. Actually, the mechanical application of the HP filter to actual output is unlikely to result in accurate estimate of potential output; it

⁶ Using $\lambda=1600$ seems to have been a de facto 'industry standard', although the choice as we have seen is based on a prior value of the ratio of the variance. Of the cycle to the variance of the trend. Baxter and King (1999) have shown that a value of around 100 for annual data is much more reasonable. They arrive at the value by visually inspecting the transfer function of the HP filter for annual data and comparing it to a band pass filter. Correia, Neves and Rebelo (1992), Cooley and Ohanian (1991) suggest a value of 400.

⁷ One of the principal limitations of the HP filter is that the analysis is purely historical and static (closed domain). The filter may have misleading predictive outcome when used dynamically since the algorithm changes (during iteration for minimization) the past state (unlike a moving average) of the time series to adjust for the current state regardless of the size of λ used, see Raven and Sola (2001), Raven and Uhlig (2002)

may fail to accurately disentangle the relative incidence of permanent and temporary shocks at the end of the sample period, given that the persistence characteristics of real shocks to output cannot be determined⁸. We, therefore, construct a range of output gap measures using different segmented paths for λ (namely, $\lambda = 1600$ from 1985 Q1 to 1995 Q2 and $\lambda = 200$ after 1995 Q2 to the end of the sample)⁹. For each case we compute the output gap and for each output gap we estimate the Phillips curve equation. We then compute the intermediate range forecast of inflation. The root mean square error (RMSE) will be smaller for an output gap with a varying λ than for an output gap with a constant λ

Kydland and Prescott [7] argue that an economy's rate of technological change is related to output and future policy arrangements and institutions in society¹⁰. Therefore, changes and *expected* changes in institutions may alter the nature, magnitude and volatility of economic shocks. Assuming a constant value of λ in the HP filter implicitly assumes that the relative variances of demand and supply disturbances to output are invariant. If, for example, the variance of aggregate demand shocks is close to the variance of the aggregate supply shocks, the HP filter will not be able to distinguish the shocks. In India the structure of the economy in the 90s has been extensively reformed with institutional changes occurring in the financial sector, government regulations and so on.

We rewrite the minimization problem of the equation (1) as

$$\text{Min} \quad = \sum_{t=1}^T (y_t - y_t^e)^2 + \sum_{t=1}^T \lambda_t [(y_{t-1}^g - y_t^g) - (y_t^g - y_{t-1}^g)]^2 \quad (5)$$

⁸ Our view is that a 5 % cyclical component change along with a 1/8 % growth component change per quarter were moderately large, but would serve as stylized thresholds, If we substitute these numbers to equation (3) $q = 1600$.

⁹ SHAZAM code is used. See Appendix 1 for a recommended choice for λ .

¹⁰ The increase in paid labor over the last few decades has gradually changed people's vacation habits, at least in some Scandinavian countries. It has become customary to spend a week of the annual holiday in the winter and this in turn has affected output and consumption in the first quarter of the year. Yet, another example may be increasingly efficient use of capital and just-in-time production techniques.

$$\lambda_t = 1600 \quad \forall_t \in (1985Q1-1995Q2)$$

$$\lambda_t = 200 \quad \forall_t \in (1995Q3-2009Q4)$$

To minimize (5) we set the parameter λ_t ranging between 200 and 1200 in increments of 100. Since the basic application of the estimated output gap is to help forecast inflation, a natural way to evaluate which output gap measure is to be preferred is to evaluate how well each measure forecasts inflation. Therefore we estimate Phillips curve equations with alternative estimates of the output gap and compare their out-of-sample forecasts of inflation.

We estimate the Phillips curve equation of the form:

$$\pi_t = \alpha E_t \pi_{t-1} + \beta (y - y^g)_{t-1} + \mu_t \quad (6)$$

where π_t quarterly inflation rate is defined as ΔP_t where P_t is the natural logarithm of the CPI excluding credit. The first term on the RHS is expected inflation measured by survey data which is a *proxy* for expected inflation. The survey measures expectations of the inflation that will occur in the next quarter. The second term on the RHS of equation (3) is the output gap and μ is the Gaussian error term. We use the Box-Cox transformation method¹¹ to estimate equation (3). All power transformation weights ω are identical for all variables. The output gap has non-positive values therefore it is not transformed. We correct for first order serial correlation. The estimates of the power weight and the first-order autocorrelation parameter are simultaneously estimated using the Savings-White method in SHAZAM. The divisor of N instead of $N - K$ is used when the variance-covariance matrix is estimated. The regression includes a constant term although the Phillips Curve does not include

¹¹ The Box-Cox procedure is chosen to choose among alternative functional forms in which theoretically-consistent homogeneity and symmetry restrictions cannot be imposed. For strictly positive value of y , the Box-Cox method defines y

$$y(\phi) = \frac{y^\phi - 1}{\phi} \quad \phi \neq 0 \quad \text{(i)} \\ = \log(y) \quad \phi = 0$$

Given the definition, the weakest assumption employed in transformation method is that, for some ϕ and \mathfrak{R}

and some $k \times 1$ vector $\beta \in \mathfrak{R}^k$ and $E[(y(\phi))] = X\beta$. By itself assumption (i) is not enough to recover $E(y | X)$ as a function of $X\beta$ and ϕ . This is not to say that ϕ and β cannot be estimated under (i),

See Amemiya and Powell (1981), also Hinkley and Runger (1984).

one.¹² Before we proceeded to test the model, we subjected the data series - output gap and inflation - to a unit root test to determine stationarity. By applying the ADF test to the series of the first order difference, we observed that both series become stationary i.e., integrated of order I (1). Details (including Correlogram) are not included here.¹³

Table 1: Unit root test

Variables	ADF	First difference
Δ Output gap	-2.314 (3)	-5.443(2)
Δ Inflation	-2.671(3)	-6.423(1)
Akaike Information Criterion	7.342	
Schwarz Criterion	7.241	

The critical value at the 5% % level is - 3.615

The model is estimated from from1998 Q2 to 2009Q2. We had to include data from the periods of high inflation to fit the curve. In recent periods, inflation has not been high which made the correlation with other variables such as the output gap very weak. The Phillips Curve cannot be tested without including observations of long - lasting changes in the inflation rate because inflation expectations cannot be observed without errors. Estimation results are reported in table 2.

Four regressions of the principal tests are reported. Model 1 is the Phillips Curve where the output gap is computed using the HP filter with

¹² What we have done here involves deriving the NAIRU directly from estimates of Phillips curves and then using Okun's law - which relates the output gap to the unemployment gap (the actual unemployment rate minus the NAIRU) - to estimate potential output. These multivariate approaches are reasonably simple and make intuitive sense. The only thing it requires that the specification of the Phillips curve is correct. For example, the model needs to correctly characterize the relationship between the unemployment rate gap and inflation dynamics while taking into account how inflation expectations are formed, and it should not leave out any other variables that have an impact on inflation.

¹³ Interested readers may consult the authors It is convenient to use the autocorrelations which are the Auto covariances normalized by dividing the variances.. $\tau_s = \tau_s / \tau_v$ $-1 \leq \tau_s \leq 1$ $s_0 = 1, 2 \dots$ If we plot τ_s $s = 1, 2, \dots$ we obtain ACF (Auto Correlation Function) or Correlogram.

$\lambda = 1600$ to estimate the trend. Models 2, 3, and 4 are similar but the output gaps are computed using the HP filter with non-constant λ 's. These λ 's are $\lambda = (1600, 200)$

$\lambda = (1600, 600)$ and $\lambda = (1600, 1200)$, respectively. In model 1 (i.e., when the output gap is measured by the HP filter with $\lambda = 1600$), the parameter α is different from one. Similar results are obtained from model 3 and model 4. These models are inadequate in the sense that the estimated parameters are not consistent with theory (Phelps and Friedman natural rate hypothesis). In model 2 (with $(\lambda = 1600, 200)$), the parameter α is different from unity. The results implying trade-off between inflation and output gap in the long run and are consistent with theory. The errors in all these models are indicated by DW statistics. The values of γ are close to zero which suggests that the models are non- linear.

Table 2: The Phillips Curve

$$\pi_t^\gamma = \delta + \alpha E_t \pi_{t-1}^\gamma + \beta (y - y^s)_{t-1} + \mu_t$$

Model 1	Model 2	Model 3	Model 4		
HP	HP	HP	HP		
$\lambda(1600)$	$\lambda(1600,200)$	$\lambda(1600,600)$	$\lambda(1600,1200)$		
δ^* (0.0001)	-0.28 (0.0026)	-0.40	-0.26 (0.000)	(0.0002)	(0.0008)
α^{**}	0.81 (0.1341)	0.93 (0.6812)	0.79 (0.2811)	0.80 (0.1113)	
β^*	0.28 (0.0001)	(0.0000)	0.22 (0.0001)	(0.0001)	0.19 0.20
γ	0.01	0.03	0.03	0.02	
ρ	0.22	0.24	0.23	0.21	
R^2	0.87	0.78	0.77	0.76	
LLF	-34.11	-34.28	-33.88	-35.23	
DW	3.11	2.44	2.22	2.28	
RMSE (3)	0.2333	0.2141	0.2482	0.2545	
RMSE (6)	0.2482	0.1882	0.2765	0.2940	
RMSE (9)	0.2333	0.1975	0.2057	0.2433	

ρ is the autocorrelation parameter
 γ is the transformation parameter
 LLF is the log of the likelihood function
 RMSE is the root mean square error. The numbers within parenthesis are the forecast horizons.
 *Denotes the p value of the t statistics for testing $H_0 : \delta = 0$ $H_0 : \beta = 0$
 **Denoted the p value of the Wald statistics $H_0 : \alpha = 1$

Next we use the Phillips curve to compute rolling forecasts of inflation. First, we use the model's estimate for the sample 1998 Q2 to 2004 Q2 to compute out-of-sample forecasts for the next three quarters 2004 Q3, 2004 Q4, 2005 Q1. We report the RMSE for each model. Second, we add the observations 2004 Q3, 2004 Q4, 2005 Q1, re estimate the models (not reported) and compute the six quarter out-of-sample forecasts (i.e., 2005 Q2, 2005 Q3, 2005 Q4, 2006 Q1, 2006 Q2, 2006 Q3). Finally we add these observations to the sample, re estimate all models and compute the nine quarter out-of-sample forecasts (i.e., we forecast inflation for 2006Q4 to 2009Q4). The results indicate that the inflation forecast using the output gap computed with a constant λ ($\lambda=1600$) is associated with higher RMSE than those resulting from varying λ . Therefore, we conclude that our technique yields relatively a better estimate of the output gap than those obtained from the traditional HP filter with a constant λ [10-19]

Conclusion

The assumption that the HP filter's smoothing parameter λ should be constant is not tenable. Therefore, we should allow for the parameter to change over time so that its value reflects the nature and magnitude of supply and demand shocks. Analysis is purely historical and static (closed domain). The filter will have misleading predictive outcome when used dynamically. Determining the exact value of λ still remains a problem, though. For better or worse, we cannot escape the need for information on output gaps so that we can forecast the future path of inflation and evaluate the current setting of our monetary policy instruments. However, we also need to recognize that because measures of potential output and output gaps are so uncertain, we must always be aware that they might be providing misleading signals as to the future course of inflation and the appropriateness of the stance of policy. In assessing whether there is a slack in the economy, we should look not only at estimates of output gaps but also at a range of indicators drawn from the labor, product, and financial markets so as to provide us with a better perspective on the balance of supply and demand in the economy.

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Appendix 1

The Appendix concludes with an optimal value for λ of 1600. The question is how is the parameter should be adjusted by a researcher confronted with data which does not come in at quarterly intervals. Consider this question from a frequency domain perspectives.

- Let the transfer function of the HP filter be given by

$$h(\omega; \lambda) = \frac{4\lambda [1 - \cos(\omega)]^2}{1 + 4\lambda [1 - \cos(\omega)]^2} \quad (1)$$

- Expressed as such altering λ is similar to altering a cut-off point in a high-pass filter.
- Let $h(\omega; \lambda_1)$ represent the ‘benchmark’ filter for quarterly data and let $h(\omega/s; \lambda_s)$ be the filter for data with sampling frequency s , where s is the frequency compared to quarterly data, i.e., $s = 1/4$ for yearly data and $s = 3$ for monthly data. We would then want
- This implies that we would like to choose a value m to fit

$$\lambda_s = s^m \lambda^1 \quad (2)$$

Consider a marginal change in s around a neighborhood’s $= 1$. For a suitable m , it should be the case that

$$\frac{d}{ds} h(\omega/s; \lambda_s) \approx 0 \quad (3)$$

where the differential operator is the total derivative of the alternative frequency filter representation with respect to s . That is,

$$\frac{d}{ds} \left(\frac{4\lambda [1 - \cos(\omega/s)]^2}{1 + 4\lambda_s [1 - \cos(\omega/s)]^2} \right) \quad (4)$$

- For any (ω/s) set chosen, (4) can be solved for the parameter m as

$$m(s/\omega) = \frac{2 \omega/s}{1 - \cos(\omega/s)} \quad (5)$$

which shows that if m is chosen correctly, $m(s, \omega)$ should remain constant over any ω frequency chosen. For $m(1, \omega) = m(s, \omega/s)$ over $0 \leq \omega \leq \pi/5$) optional choices include

$$\omega = 0 \Rightarrow m = 4$$

$$\omega = \frac{\pi}{20} \Rightarrow m = 3.992$$

$$\omega = \frac{\pi}{10} \Rightarrow m = 3.967$$

$$\omega = \frac{\pi}{5} \Rightarrow m = 3.868$$

- According to the list, a value of $m = 4$ is an appropriate choice in order to make the transfer function (1) invariant to the frequency of the data undertaken.