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RESEARCH ARTICLE

A Mixed Integer Programming Solution for Transmission Switching in Power System

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Abstract

The ultimate objective of the transmission system is to deliver electric power reliably and economically from generators to loads. Co-optimize transmission topology and generation dispatch could be a viable way to minimize generation cost. Transmission network topology is changed by using transmission line switching. Transmission Switching is also beneficial for both operator and consumer in the restructured electric power industry. Mathematical model is formulated by representing transmission line switching by binary variable and power flow in the lines and generation by continuous variable. Hence the formulated problem is a MILP (Mixed Integer Linear Problem). Formulated MILP problem solved by using GLPK solver of optimization with PYOMO (Python Optimization Modeling Objects) language. Branch and Bound method of MILP is used by GLPK solver. In the study a standard IEEE 30-bus system is tested and significant savings in system dispatch cost and other market benefits achieved.

Keywords: *MILP, Restructured Power System, Transmission Switching's.*

Introduction

Demand of electrical power is increasing with time. Growing demand for electric power seems to necessitate addition of new transmission lines, but problem to building these new lines are often extremely high. At the same time the recent trend is to push for a "smarter" bulk electric grid, one that is more flexible and controllable. In transmission network transmission switching is a straightforward way to make better use of the existing system and meet growing demand with existing infrastructure. Fundamental physics that govern electrical networks stated that the physical characteristic and the flow on each line impact flow on all lines. It is therefore possible to remove a transmission line in an order to improve the output of the system. However switching of transmission line is a common practice done with a mature technology. Transmission lines are open and closed by using circuit breakers.

Transmission switching has been used as a control method for problems such as, loss and/or improving system security [3], cost reduction [4], over- or under-voltage situations [8], line overloads [9] and combination of these [2, 5]. Bacher et al. [2] presented a method to reduce the current in overloaded line by using Network topology optimization with security constraint. Schnyder et al. [3] presented a method for control n-1 security by using an optimal power flow and switching concept. Schnyder et al. [4] gave a method for security enhancement using an optimal switching power flow with Double switching operation (outage-control action). Rolim et al. [5] gave reviews of several publications which were presented on corrective switching algorithm in transmission system. O'Neill et al. [7] presented a paper on Dispatch able transmission in regional transmission organization markets and beneficial effect on revealed surplus was obtained. Shao et al. [8] presented a method with Corrective switching algorithm for relieving overloads and voltage violations. Fisher et al. [11] gave a formulation for optimal transmission switching to reduce dispatch cost of network.

Transmission is traditionally characterized as a static system with random outages over which the system operator dispatches generators to minimize cost. Power flows from generators to loads along the lines in these networks is accordance with the laws of physics, which dictate that power flow along all lines in proportion to the electrical characteristics of those lines. Therefore, the throughput of the system can be increase by remove a line. The Northeast Power Coordinating Council includes "switch out internal transmission lines" in the list of possible actions to avoid abnormal voltage conditions (Northeast Power Coordinating Council, 1997). In addition, system operators have procedures in place to close lines quickly in case of emergency.

This paper is examinations the potential of switching to increase the economic efficiency of power system dispatch by embedding switching decisions within the system dispatch model and optimizing the transmission topology simultaneously with generator output is done. To embed these decisions, we incorporate transmission switching into the standard optimal power flow program used to dispatch generators in power networks. Adding transmission switching to this problem introduces non-continuous variables (e.g. a line is either open or closed), which are modelled with binary variables. The result is that the linear OPF model becomes a mixed-integer linear problem (MILP). The optimization problem used to solve the dispatch problem uses generation cost (and when appropriate, consumer willingness to pay) as the objective function. This choice is based on economic theory, which says maximizing social welfare—the difference between consumer demand and generator cost curves—is desirable because it results in efficient production and trade. In this case demand is fixed, maximize social welfare is equivalent to minimizing generation cost. This objective is a valid one for both regulated systems, where generation dispatch is a centralized process in which all operating costs are known, and for restructured market system (market-based systems). In restructured market system dispatch is determined by a centralized grid operator who takes bids from generators and depends on those generators to operate as directed. Cost savings from transmission switching can be realized in both regulated and market systems. The difference lies in payments and cash flow. In regulated systems, generators, transmission lines and loads are all owned and managed by the same entity, and thus money is not really changing hands: generators are essentially "paid" the cost of operating (or rather the central utility is paying the fuel costs, worker salaries, etc., directly). In a market based system, however, generators are not paid their costs, but rather the nodal prices at the point of interconnection. The nodal price at each node (or bus or connection point) in the system is the value of a marginal change in demand at that node. This value is easy to get from the optimization solution used for dispatching the system: nodal prices are simply the dual variables on the power balance constraints. In markets, therefore, payments to generators can be different than their cost. In other words, minimizing generator cost is guaranteed in the solution, but there is no indication of what happens to nodal price, and thus payments to and from market participants. To analyze these changes, we must look at nodal prices as well. The paper is organized as follows. Section II presents the MILP formulation for the transmission switching problem. Section III discusses results using new the formulation on the IEEE 30-bus system, and Section IV concludes. Nomenclatures are given in appendix.

Problem Formulation

Currently, power system are Dispatched, or generation is scheduled to meet demand, by using an optimization model called an optimal power flow (OPF). The OPF minimize generation cost (or minimize generator bids, or maximize social welfare, depending on the market structure) subjected to physical constraints of the system and Kirchhoff's laws governing power flow. AC Line Flow Equations (Kirchhoff's laws) given by equation (1) and (2)

$$P_k = V_n V_m G_k \cos(\theta_n - \theta_m) + V_n V_m B_k \sin(\theta_n - \theta_m) \quad (1)$$

$$Q_k = V_n V_m G_k \sin(\theta_n - \theta_m) - V_n V_m B_k \cos(\theta_n - \theta_m) \quad (2)$$

Alternating Current Optimal Power Flow (ACOPF) optimization problem is, however a very difficult problem to solve since it is a non-convex optimization problem with constraints that involve trigonometric functions. It is common in academic literature as well as in the industry, to use a linear approximation of the ACOPF problem. The first simplification that is made is that the conductance (G_k) is assumed to be zero. The second assumption concerns the voltage variables, V_n and V_m equal to 1. The next assumption comes from the fact that the angle difference between two buses is typically very small. The last simplification is that the reactive power (Q_k) is ignored. All of these assumptions then produce the linear approximation of the AC optimal power flow (ACOPF) problem, which is known as the Direct Current Optimal Power Flow (DCOPF) problem. The DCOPF problem is a linear program so it is much easier to solve than the non-convex ACOPF problem. With these assumptions, (2) are ignored and (1) modifies as (3) given below:

$$B_k(\theta_n - \theta_m) - P_k = 0 \quad (3)$$

Making the objective function more robust would ensure all appropriate operating costs were being considered in the solution.

Traditional OPF Formulation

Objective function:

$$TC = \sum_g C_g P_g \quad \$/h \qquad (4)$$

Subjected to

$$\theta_n^{max} \le \theta_n \le \theta_n^{max} \qquad \forall n \qquad (5)$$
$$P_g^{max} \le P_g \le P_g^{min} \qquad \forall g \qquad (6)$$
$$P_k^{min} \le P_k \le P_k^{max} \qquad \forall k \qquad (7)$$
$$\sum_g P_g - \sum_k P_k - \sum_d P_d = 0 \qquad \forall n \qquad (8)$$
$$B_k(\theta_n - \theta_m) - P_k = 0 \quad \forall k \text{ with endpoints n, m} \quad (9)$$

The constraints represent physical operating limits of the network. Voltage angle limits are imposed by (5), Constraint (6) limits the output of generator at node to its physical capabilities, and (7) limit the power flow across line at node. Power balance at each node is enforced by (8), and Kirchhoff's laws are incorporated in constraint (9). All variables and parameters are defined in the appendix. This formulation is very general, and could be expanded to include any other constraint or objective function that would represent the system more accurately.

This formulation is modified to represent transmission switching, a vector of binary variables is added that represent the status of each transmission line. In this new formulation, each line is assigned a binary variable z_k, that represents whether the line is included in the system (the circuit breaker on that line is closed) or not (circuit breaker is open). z_k is the binary variable representing the state of the transmission line; a value of one reflects a committed or closed element and a value of zero reflects an uncommitted or open element. When a transmission line is opened, two things happen: no flow is allowed over the line, and that line needs to be removed electrically from the network, so that it no longer impacts flows on other lines. If the flow were only limited to zero, the network model would continue to enforce the laws relating every flow to every line in the network, and would seriously constrict flow on the whole network. Put another way, this would be equivalent to having one line in a network with an extremely small capacity constraint: almost no power could flow on that one line and that limit would affect flow on all other lines as well. Therefore, to model transmission switching correctly, several constraints must be modified. First, the minimum and maximum transmission capacity constraints are multiplied by the binary variables in the constraint defining transmission flow limits. This limits the flow over that line to zero if the variable z_k is zero. Because of the constraint (9) represents Kirchhoff's laws, the formulation of the problem is more complicated than simply limiting the power flow to P_k^{min} or P_k^{max} times the binary variable. M_k, listed in (15) and (16), is referred to as the "big M" value. When the binary variable z_k is one, the value of M_k does not matter; when the binary variable is zero, the value of M_k is used to ensure that (15) and (16) are satisfied regardless of the difference in the bus angles. Power flow in line k (P_k) is zero when z_k is zero so M_k must be a large number greater than or equal to $B_k * (\theta_n^{max} - \theta_m^{min})$. It is computationally favourable to have M_k be as small as possible; thus, set it equal to $B_k * (\theta_n^{max} - \theta_m^{min})$.

Without this adjustment to the power flow equations, the buses that are connected to this opened transmission element would be forced to have the same bus angle. Forcing the buses' angles to be the same is incorrect as the element is no longer present. For the situation where there are two parallel transmission lines, if the program removes one line, the other would be forced to have a zero power flow value without this adjustment to the power flow equations. Modified OPF with transmission switching is given below.

OPF Formulation with Transmission Switching

Objective function:

$$TC = \sum_g C_g P_g \quad \$/h \qquad (10)$$

Subjected to

$$\theta_n^{max} \le \theta_n \le \theta_n^{max} \qquad \forall n \qquad (11)$$
$$P_g^{max} \le P_g \le P_g^{min} \qquad \forall g \qquad (12)$$
$$z_k P_k^{min} \le P_k \le z_k P_k^{max} \qquad \forall k \qquad (13)$$
$$\sum_g P_g - \sum_k P_k - \sum_d P_d = 0 \qquad \forall n \qquad (14)$$

$$B_k(\theta_n - \theta_m) - P_k + (1 - z_k)M \ge 0 \quad \forall k \text{ with end points n, m} \qquad (15)$$

$$B_k(\theta_n - \theta_m) - P_k - (1 - z_k)M \le 0 \quad \forall k \text{ with endpoints n, m} \qquad (16)$$

$$\sum_k (1 - z_k) \le T \qquad \forall k \qquad (17)$$

In this formulation added a lines upper bound (LUB) constraint (17) that can limit the number of open lines in the optimal network topology. This constraint is used to gain understanding about the effects of changing the network topology.

To understand how the transmission switching effecting the restructured power system network, payments to generators and payments from loads, based on a nodal marginal price are considered. These payments are the location marginal price (LMP) or nodal price, congestion rent, generation rent, generation revenue, and load payment. The locational marginal price (LMP), is the marginal value of energy at a given location in the network, and is calculated as the dual variable of the power balance constraint (8). Total system cost is the sum of all the costs in the system to meet the load, and is referred to as generation cost within this paper. In the presented model, this comprises variable generator cost. Generator revenue is the amount generators are paid based on nodal pricing, LMP times amount produced. Generation revenue is the sum of all generator revenues. Generator rent, then, is the difference between revenue and cost for an individual generator and generation rent is the sum of all generator rents. Load payment is the sum of all individual load payments, which is the nodal payment or LMP time's amount consumed. Congestion rent can be defined as the difference in LMPs across a transmission element times its power flow and can be given as:

Congestion Rent = $(LMP_m - LMP_n) * P_k$ where k^{th} line connected b/w n & m (18)

Test Network Results and Analysis

The formulation presented above was used to dispatch a common engineering test case: the IEEE-30 bus system. The transmission switching problem was written in PYTHON language and solved by PYOMO with GLPK solver. The generator costs modelled as linear, assumed resistance and shunt capacitance of lines were zero, ignored losses and reactive power. The traditional OPF and modified OPF with switching were tested on the case to assess potential benefits of Transmission Switching. Also, this method was run on the system with various load profiles to highlight the benefits of system flexibility. The IEEE 30-bus test case was used to test and analyze the above formulation. Data for the test system was downloaded from University of Washington Power System Test case Archive and transmission line characteristic and generator variable costs were taken from the network as reported in [16]. The system consists of 30 buses, 41 transmission lines, and 6 committed generators with a total capacity of 435 MW. The total 29 load buses are connected with a total consumption of 373.40 MW. When no transmission lines are opened, the system cost of meeting the demand is $936/h which is find out with traditional optimal power flow.

To gain insight on how these lines are affecting system dispatch cost, limit on the number of open lines by including the lines upper bound (LUB) constraint.

Allowing only one line to open (T=1) results in K_1 = {line 35}, meaning line 35, connecting bus 25 to 27, opens. By opening the line changes the power output of four generators at buses 1, 2, 8 and 13 (see table I). Generator at bus 8 decrease output, and the generators at buses 1, 2 and 13 increases output. As can be seen in Table I, the increased generators are less expensive than the decreased generator (at bus 8).

Table 1: Changes in generator output after opening line 35

Generator Bus	Variable Cost [$/MWh]	Output [MW]
1	2	+3.84
2	2	+23.84
8	6	-34.56
13	5	+6.88

Repeating this analysis for T= {1,..., 6}, it is identified that the sequence of subsets, K_1, and the objective function values for each of these topologies. For $|K_T| \leq |K_{T+1}|$ it is not necessarily true that, $K_T \subset K_{T+1}$, see in particular range from T=5 to T=6. In other words, these sequentially-found subsets are not necessarily subset of one another. Also, for each additional open lines, the system cost decreasing rate (see table II).

Table 2: Sequence of line opening with 30 bus systems

Number of open lines allowed (T)	Open lines	System dispatch Cost	percentage saving in TCj
0		$936	
1	35	$819	12.50%
2	9,35	$778	16.88%
3	9,12,35	$770	17.73%
4	9,12,25,35	$768	17.94%
5	9,12,25,26,35	$767	18.05%
6	9,12,24,26,29,35	$767	18.05%
Without LUB (Case)	9,12,24,26,29,35	$767	18.05%

This implies two things about this network: one is that a small number of lines have a large impact on dispatch cost, and a large number of lines have a small impact (see table II). The other is that improving system cost by opening lines is not a linear process; serially opening the next line that provides the most improvement will not necessarily produce the optimal network (see fig. 1). Thus, to gain the majority of economic benefit from OTS a large number of lines need not be switched.

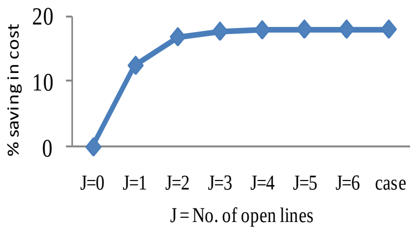

Fig. 1: System dispatch cost with no. of lines opens

Fig 2 displays the fluctuations in generation cost, generation revenue, generation rent, and load payment for various solution to the transmission switching problem for varying values of T (T={0,……, 4}). The value in figure are displayed as percentage of results from the T=0 case, i.e. the percentage value reflect the specific case's value devided by the T=0 case vaue. For example, generation rent is $165/h for T=0 and 140% of that, or $232/h, for T=3. Because these are percentages, the values shown do not add up in the way the actual values do; thus, the percentage value for generation rent plus cost does not add up to the percentage value for the generation revenue.

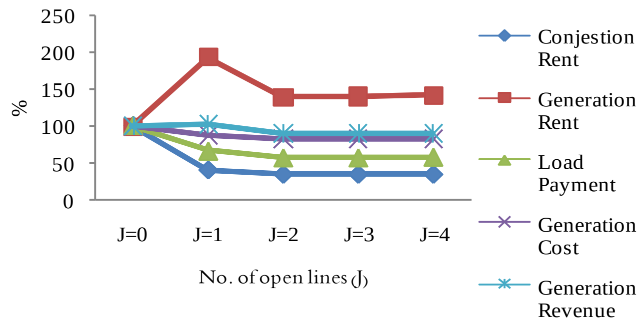

Fig. 2: Generation rent, load payment and congestion rent for various solutions

The case where T=0 has a generation cost of $936/h, generation revenue of $1102/h, generation rent of $165/h, congestion rent of $1556/h, and load payment of $2657/h. Note that the congestion rent is unusually high; typically congestion rent is 5%-10% of generation cost. The best found solution reduces operating cost to $767/h, which is 81.94% of the T=0 cost of $936/h. Results from Fig. 2 indicate that for the majority of cases both the generators and the consumers are benefiting in comparison to the case with no open transmission elements. The generation rent is typically higher while the load payment is almost always lower than in the T=0 case. Congestion rent, in contrast, is generally lower than those calculated in the case. One key result is that generation revenue, congestion rent, load payment, and generation rent fluctuate dramatically as changes.

Table 3: Operating condition of transmission network during a day

Time Period	12-4 AM	4-8 AM	8-12 AM	12-4 PM	4-8 PM	8-12 PM
Load	60%	73%	100%	74%	86%	96%
No. of lines open	6	8	6	5	7	9
No. of generators running	3	3	6	3	4	5

To get actual optimal solution on different loading scenario, optimal power flow must run with changing load profile (in table III shown % load for 24 hours). To simulate different loading scenarios, load curve of a day considered. Table III shown the operating condition of transmission lines and generators of the system for a given load curve. Different pattern of loading benefit from different topologies; the optimal network for one load scenario is different from optimal network of another.

These results indicate that a transmission network optimized for one particular pattern of load on a network is not necessarily optimal for another. Thus, allowing decisions about network topology to be made based on real-time system conditions (or daily or weekly forecasts) can result in a lower-cost dispatch than using a static network optimized for a multi-period forecast.

It may not be perfectly clear why transmission switching can provide savings with well planned networks. There are some reasons by which it can be justified. First, we used the IEEE 30-bus test case as it is a standard IEEE test case and have found significant results with this network. Second, it is important to understand that the OPF equations ensure that load demand is always satisfied no matter the configuration of the network (14); thus, there is no load shedding. Last, and more importantly, transmission planning is supposed to determine the best network configuration by examining the total benefits over many future years. Thus, this is a long run problem, which is different from transmission switching, which deals with the short run problem of finding the best topology for a specific hour. This is one key reason why transmission switching can provide benefits even in well planned networks. Transmission switching can also be beneficial since transmission planning is very difficult based on the great uncertainty of future network conditions.

Conclusion

In this paper a method has been described to determine optimal transmission network topology and change in generation output to meet the demand with minimum dispatch cost. From the results it can be said that large improvements in system dispatch cost can be achieved by switching off certain transmission lines under specified system conditions and it is beneficial for both operator and consumer in restructured market system.

An improvement in system cost of 18.05% is achieved in the IEEE 30-bus test network when 6 lines were open. Generation Rent and Load Payment also improved by transmission switching. Most of the improvement in dispatch cost is realized by changing the status of a few lines. A transmission network optimized for one particular pattern of load on a network is not necessarily optimal for another. Thus, allowing decisions about network topology to be made based on real-time system conditions (or daily or weekly forecasts). Real-time (or close to real-time) control can result in more efficient transmission topologies than static ones, even if the static ones were originally designed to be optimal.

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Appendix

Nomenclature:

n, m	No. of Nodes
k	No. of Lines
g	No. of Generators
d	No. of Loads
T	Number of lines allowed to opens θ_n Voltage angle at node n
TC_T	Total system cost with T lines open
C_g	Cost of generating electricity from generator g
B_k	Electrical Susceptance of line k
P_k, P_g, P_d	Real power flow to or from line, Generator or load
P_k^{max}, P_k^{min}	Maximum and minimum capacity of line
P_g^{max}, P_g^{min}	Maximum and minimum capacity of generator.
$\theta_n^{max}, \theta_n^{min}$	Maximum and minimum voltage angle at node n and m
V_n, V_m	Voltage magnitude at buses n and m respectively
z_k	Binary variable indicating state of transmission line
M_k	Large number