

RESEARCH ARTICLE

Monetary Policy Rules Evaluation Using a Forward Looking Model for Romania

Ana-Maria Săndică*, Alexie Alupoaei

Bucharest Academy of Economic Studies, Romania.

*Corresponding Author: Email: anamaria.sandica@gmail.com

Abstract

The starting point of this paper was a small structural model, the next step was to estimate the parameters and then to evaluate the performance of alternative rules treating structural parameters as fixed and known. Performance evaluation was done through a policy loss function with three inputs. The first input is a set of three weights representing the relative importance of the central bank to stabilize inflation, output and interest rates. The first conclusion is that the rules that recorded the lower values on loss function are the Optimal Taylor with interest rate smoothing (T) and Full state rule (FS). The latter rule is best when the weight on output gap is smaller than 0.25.

Keywords: *Inflation targeting, Interest rate policy, Optimal monetary policy, Taylor's rule*

JEL: E52, E58, P20

Introduction

In 1998, John Taylor [1] underlined that “researchers first build a structural model of the economy, consisting of mathematical equations with estimated numerical parameter values. They then test out different rules by simulating the model stochastically with different policy rules placed in the model. One monetary policy rule is better than another monetary policy rule if the simulation results show better economic performance.”

The starting point of this paper was a small structural model, the next step was to estimate the parameters and then to evaluate the performance of alternative rules treating structural parameters as fixed and known. Performance evaluation was done through a policy loss function with three inputs. The first input is a set of three weights representing the relative importance of the central bank to stabilize inflation, output and interest rates. The second input in the loss function is structural error covariance matrix. For rules with fixed coefficients, we used the computational algorithm Klein to obtain the reduced error covariance matrix out of the covariance matrix of structural errors, structural parameters and coefficients of the policy rule. The last input is the state transition coefficients matrix.

In the theoretical background are presented how the coefficients of a policy rule are computed when the structure is “backward-looking” comparing with “forward-looking” models. Also it is presented the structural model that underlies the analysis and explains how is use the Klein algorithm to solve it and compute policy loss. In empirical section it is presented the results of the policy evaluation for fixed coefficient rules. The last section concludes.

Literature Review

In literature there are two types of rules for monetary policy, the simple rules tools such as Taylor [1] and others and targeting rules. The first type of simple rules represents an instrument of monetary policy based on economic status. Examples of these rules are Taylor [1]. The most know rules is formulated by Taylor [1] and the assumption is that monetary authorities should raise interest rates by one and a half whenever inflation deviates from target with its point, and should increase by half point interest rate for each percentage point increase in the output gap. Simplicity Taylor rule has become the reference for the discussion of monetary policy. Several articles [2], have shown that the rule is consistent with stability but its optimality depends on the parameters of the economy. For

the "targeting rule" approach, the assumption is that the central bank defines a loss function. In order to minimize this function, a set of vector of target variables and target levels is assumed. In the literature, most frequently appear flexible inflation targeting strategy, which was developed in the work developed by Taylor [3]. Backward-looking models have been supported by both academic economists and monetary authorities, and their application in several research studies is frequent. Models with forward-looking expectations tend not to fit the data well, unlike the models proposed. Monetary policy is optimal,

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - \pi_t) + u_t \tag{1}$$

$$\pi_t = \beta y_t + \alpha \pi_{t-1} + v_t \tag{2}$$

$$r_t = \theta_1 y_{t-1} + \theta_2 \pi_{t-1} + \theta_3 r_{t-1} + \theta_4 y_{t-2} + w_t \tag{3}$$

The first equation is a backward looking IS curve, the second equation is a backward looking Phillips curve which implies that the inflation tends to rise when the output exceeds its steady state value. The last equation explains how the central bank adjusts the nominal interest rate in

to some extent, to its history, or in other words, to its backward-looking behaviour [4-6].

Theoretical Background

Optimal policy is characterized by the matrix Riccati equation when the state transition equation is linear and the bank's objective function is quadratic. The backward iteration of the Riccati equations shows that optimal policy is a fixed-coefficient rule. The economy is built around three equations for output, inflation and interest rates.

response to changes that are in the economy. The set of values for the parameters of the feedback equation is a monetary policy. The structural shocks from the three equations are assumed to have zero mean and to be serially uncorrelated. The reduced form could be written as:

$$Z_t = AZ_{t-1} + Cr_t + U_t \tag{4}$$

Where $Z_t = (y_t, \pi_t, r_t, y_{t-1})'$, $U_t = (\eta_{1t}, \eta_{2t}, 0, 0)'$, $\eta_{1t} = d(u_t + bv_t)$, $\eta_{2t} = d(\beta u_t + v_t)$,

$d = (1 - b\beta)^{-1}$ and where A and C are matrices given by:

$$A = \begin{bmatrix} da_1 & db\alpha & 0 & da_2 \\ d\beta a_1 & d\alpha & 0 & dba_2 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} db \\ -db\beta \\ 1 \\ 0 \end{bmatrix} \tag{5}$$

The first assumption is that the central bank chooses values for $\theta_1 \dots \theta_4$ that minimizes the loss function:

$$\Lambda = E_0 \sum_{t=0}^{\infty} \delta^t Z_t' W Z_t \tag{6}$$

Where W is a matrix (4x4) of policy weights that determine the relative importance accorded by central bank in respect with stabilization

objectives. The second assumption is that the transition is linear therefore the solution is given by:

$$r_t = \Theta Z_{t-1} + v_t \tag{7}$$

Where Θ is the vector of reaction coefficients, $\theta_1 \dots \theta_4$. The optimal value for this vector is calculated using Riccati equations. For the forward looking model, the optimal monetary policy needs to be

computed by numerical minimization of loss. Writing the variables as a first-order vector autoregression:

$$GX_{t-1} + \Phi_t \tag{8}$$

Where $X_t = (y_t, \pi_t, r_t, y_{t-1}, \pi_{t-1}, r_{t-1})'$, $\Phi_t = (\varphi_{1t}, \varphi_{2t}, \varphi_{3t}, 0, 0, 0)'$, $\varphi_{1t} = d(u_t + bv_t - bw_t)$,

$\varphi_{2t} = d(\beta u_t + v_t - b\beta w_t)$, $\varphi_{3t} = w_t$ and the (6x6) matrix G is:

$$G = \begin{bmatrix} G_{11} & G_{12} \\ 1 & 0 \end{bmatrix} \quad G_{11} = \begin{bmatrix} d(a_1 - b\theta_1) & db(\alpha - b\theta_2) & -db\theta_3 \\ d\beta(a_1 - b\theta_1) & d(\alpha - b\beta\theta_2) & -db\beta\theta_3 \\ \theta_1 & \theta_2 & \theta_3 \end{bmatrix} \quad [9]$$

$$G_{12} = \begin{bmatrix} d(a_2 - b\theta_4) & 0 & 0 \\ d\beta(a_2 - b\theta_4) & 0 & 0 \\ \theta_4 & 0 & 0 \end{bmatrix}$$

Because they are linear combinations of the serially uncorrelated structural errors, the Φ_{jt} are serially uncorrelated. The moving average representation for X_t is $(I - GL)^{-1}\Phi_t$ where L is

$$\Lambda = E_0 \sum_{t=0}^{\infty} \delta^t X_t' \tilde{W} X_t = \quad [10]$$

$$E_0 \sum_{t=0}^{\infty} \delta^t \text{trace}[\tilde{W} E_0(X_t X_t')] = \text{trace}[\tilde{W} \sum_{t=0}^{\infty} \delta^t E_0(X_t X_t')]$$

$$= \text{trace}[\tilde{W} \sum_{t=0}^{\infty} \delta^t (E_0(X_t - E_0 X_t)(X_t - E_0 X_t)' + (E_0 X_t)(E_0 X_t)')] = \text{trace}[\tilde{W}(M + N)]$$

Where \tilde{W} is a diagonal matrix (6x6), M is the discounted sum of forecast error variances of X computed at time zero when policy is set. N is the discounted sum of quadratic terms in expected departures of X from its target. Provided that the economy is on target at the time when policy is set, N=0 and the objective of the central bank is to minimize the part of Λ that involves M. The trade-off between returning the economy to its

the lag operator. The next step is to write Λ as a function of the forecast error variance of the model's variables:

target and minimizing the weighted sum of discounted error variances is happening when economy begins to move away from the target path. In this analysis I assume that N=0. The last step is derivation of a convenient expression for M. Let Ω be the (6x) covariance matrix for Φ_t and due to the fact that is serially uncorrelated:

$$E_0(X_k - E_0 X_k)(X_k - E_0 X_k)' = \Omega + G\Omega G' + G^2\Omega(G^2)' + \dots + G^{k-1}\Omega(G^{k-1})' \quad [11]$$

And

$$M = \Omega + \delta[\Omega + G\Omega G'] + \dots + \delta^k[\Omega + G\Omega G' + G^{k-1}\Omega(G^{k-1})]' + \dots \quad [12]$$

$$= (1 - \delta)^{-1}[\Omega + \delta G\Omega G' + \delta^2 G^2\Omega(G^2)' + \dots]$$

In order to minimize directly the strategy is to compute M by iterating square-bracket term in $y_t = \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t \pi_{t+1}) + u_t$
 $\pi_t = \beta y_t + \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + v_t$
 $r_t = \theta_1 y_{t-1} + \theta_2 \pi_{t-1} + \theta_3 r_{t-1} + \theta_4 y_{t-2} + w_t$

Eq. 12 to convergence and computes loss as trace. Assuming a forward looking model of form:

$$[13]$$

$$[14]$$

$$[15]$$

The first equation represents the IS curve and might be obtained by combining a linearized Euler equation that characterizes a representative household' optimal choice between consumption and saving. The presence of expected future output in IS curve is explained by the household smooth consumption behaviour. The second equation is the Phillips curve and when α_2

is null then is the curve. When the coefficient α_2 is different by zero the Eq. [2] is a new hybrid Phillips curve developed for explain inertia in the rate of inflation. The three equations above introduce two pivots of complexity, first the agents' actions depend upon expected output and inflation, secondly agents' beliefs are ration and cause changes in θ parameters.

$$\tilde{A} \begin{bmatrix} Z_t \\ E_t y_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \tilde{B} \begin{bmatrix} Z_{t-1} \\ y_t \\ \pi_t \end{bmatrix} + \tilde{C} S_t \quad [16]$$

Where $Z_t = (y_t, \pi_t, r_t, y_{t-1})'$, $S_t = (u_t, v_t, w_t)'$, and where \tilde{A} , \tilde{B} and \tilde{C} are given by:

$$\tilde{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -b & 0 & \lambda & b \\ 0 & 0 & 0 & 0 & 0 & \alpha_1 \end{bmatrix} \tilde{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \theta_1 & \theta_2 & \theta_3 & \theta_4 & 0 & 0 \\ -a_1 & 0 & 0 & -a_2 & 1 & 0 \\ 0 & -\alpha_2 & 0 & 0 & -\beta & 1 \end{bmatrix} \quad [17]$$

$$\tilde{C} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

Z_{t-1} is the vector of backward-looking variables and y_t and π_t are the forward-looking variables. The matrices \tilde{A} and \tilde{B} are decomposed using a generalized “QZ” decomposition. For any pair of

$$\tilde{A} = Q'SZ' \quad \tilde{B} = Q'TZ \quad QQ' = ZZ' = I$$

The generalized eigenvalues of the system are the ratios T_{ii}/S_{ii} where T_{ii} and S_{ii} are the diagonal elements of T and S. The number of stable eigenvalues equals the number of backward-looking variables, Klein shows that the unique solution for the backward looking variables is given by:

$$Z_t = (Z_{11}S_{11}^{-1}T_{11}Z_{11}^{-1})Z_{t-1} + LS_t \quad [19]$$

For this model a unique solution will exist if there are four stable and two unstable eigenvalues. First the algorithm chooses a starting value for Θ , using Eq.[4] to compute the reduced form and the resulting G matrix and then calculates policy loss using Eq. [10] and Eq.[12]. The second step is to calculate partial derivatives of loss with respect to each element of Θ . Because private agents respond to policy changes by changing their beliefs and actions for every change in Θ , G must be recomputed. The algorithm repeats steps two and three until it can no longer lower policy loss.

Empirical Results

The Taylor Rule proposed by John Taylor may be written as:

$$r_t = \theta_y y_t + \theta_p \pi_t + \theta_r r_{t-1} \quad [20]$$

The coefficients values from 1993 are 0.5, 1.5 and 0. One alternative of the original is a rule that

λ	a_1	a_2	b	β	α_1	α_2
0.4469	0.7215	-0.1228	0.0460	0.0305	0.4089	0.5651

Table 1 reports the policy rule that performed the lowest loss level for each set of policy objective weights that have been considered (See annexes for completed grid). Nodes on diagonal represent

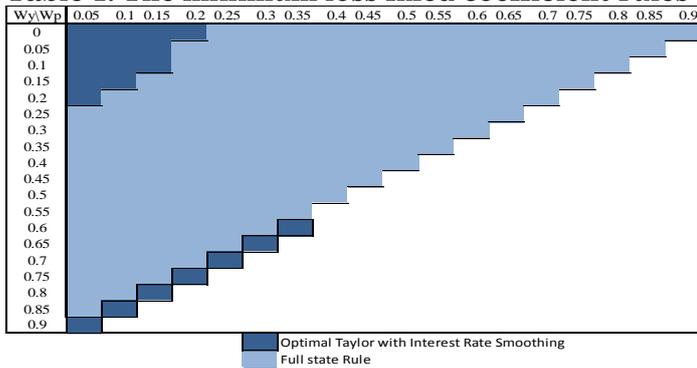
conformable matrices (\tilde{A}, \tilde{B}) there exist orthonormal matrices Q and Z and upper triangular matrices S and T such that

$$[18]$$

sets the last parameters as 0 but choose values for inflation and output gap in order to minimize the loss function. In 1999, Taylor proposed that for interest-rate smoothing the coefficient of the past interest rate to be positive. One of the critical among researches was that policy makers can react only to lagged values and not current one, in this respect Taylor rule has been updated in 1999 with lagged values. In order to assess how important it is for the Central Bank to correctly specify the state vector, a comparison between Taylor rule^[19] with lagged variables and the rule from Eq.[15] will be realized. The difference is that on latter the central bank will respond better to business cycle momentum by using two lags for output gap. When choosing an optimal value for inflation in Eq. [20] and setting the other two parameters as zero the rule is called Goodhart rule. The fact that central bank reacts to expected future inflation, therefore using expectations of inflation instead of π_t in Eq. [20] will cover this approach.

The data used is on quarterly basis, from 2000Q1 to 2011Q1: CPI (consumer price index), GDP (gross domestic production) and interest rate. The output gap and the interest rate gap are measured as the deviation from the trend, was calculated with Hodrick-Prescott filter. The inflation target variable is calculated as the deviation from the inflation target settled by the Central Bank. The coefficients obtained from the system [13]-[15] are detailed in Table1:

cases in which minimal weight was assigned to stabilize the rate of interest, above the diagonal are the cases where higher weights was assigned to the objective of interest rate smoothing.

Table 1: The minimum loss fixed coefficient rules

The first conclusion is that the rules that recorded the lowest values on the loss function are the Optimal Taylor with interest rate smoothing (T) and Full state rule (FS). The latter rule is best when the weight on the output gap is smaller than 0.25. The results are in line with other research on the Romanian economy, Murarasu (2004) who obtained that TS is best when W_y is higher than 0.1 no matter the distribution of weight across other objectives.

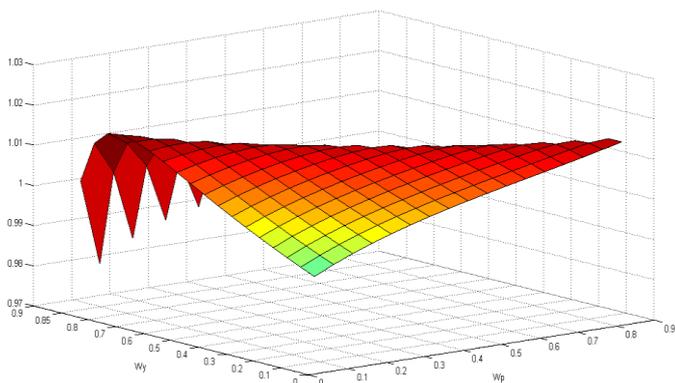


Fig. 1: Loss policy ratio optimal Taylor with interest rate smoothing vs full state rule

References

1. Taylor John B (1993) Discretion versus Policy Rules in Practice”, Carnegie-Rochester Conference Series on Public Politics, 39:195-214.
2. Ball L (1999) Policy Rules for Open Economies, in John B. Taylor, ed., Monetary Policy Rules. Chicago, Illinois: University of Chicago Press, 127-144.
3. Taylor J (2000) The Monetary Transmission Mechanism and The Evaluation of Monetary Policy Rules, Central Bank of Chile Working Paper no.87/2000, 12-23.
4. Calvo Guillermo A (1978) On the Time Consistency of Optimal Policy in a Monetary Economy. *Econometrica*, 46(6):1411-28.
5. Gelain P (2007) The Optimal Monetary Policy Rule for the European Central Bank, European Central Bank Studies.
6. Linde J (2000) Monetary Policy Analysis in Backward-Looking Models, Central Bank of Sweden WPPaper No. 114.

The figure shows that the Taylor Rule with Interest Rate Smoothing performs at all times better than the Taylor Backward Looking Rule.

Conclusions

The starting point of this paper was a small structural model, the next step was to estimate the parameters and then to evaluate the performance of alternative rules treating structural parameters as fixed and known. Performance evaluation was done through a policy loss function with three inputs. The first input is a set of three weights representing the relative importance of the central bank to stabilize inflation, output and interest rates. The first conclusion is that the rules that recorded the lowest values on the loss function are the Optimal Taylor with interest rate smoothing (T) and Full state rule (FS). The latter rule is best when the weight on the output gap is smaller than 0.25. The analysis shows that the Taylor Rule with Interest Rate Smoothing performs at all times better than the Taylor Backward Looking Rule.

Acknowledgement

This work was co-financed from the European Social Fund through Sectoral Operational Programme Human Resources Development 2007-2013; project number POSDRU/107/1.5/S/77213 „Ph.D. for a career in interdisciplinary economic research at the European standards”.

Annex 1: Matrix Error Covariance

$$\begin{pmatrix} 2.29E - 05 & -1.45E - 05 & 8.93E - 06 \\ -1.45E - 05 & 6.97E - 05 & 2.53E - 05 \\ 8.93E - 06 & 2.53E - 05 & 2.48E - 04 \end{pmatrix}$$

Annexe 2: The complete grid with the minimum loss fixed coefficients.

Wy\Wj	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9
0	0.0264	0.0283	0.0302	0.0320	0.0339	0.0357	0.0374	0.0392	0.0409	0.0427	0.0444	0.0460	0.0476	0.0492	0.0506	0.0518	0.0527	0.0529
0.05	0.0259	0.0278	0.0296	0.0315	0.0333	0.0351	0.0368	0.0386	0.0403	0.0420	0.0437	0.0453	0.0468	0.0482	0.0495	0.0505	0.0508	
0.1	0.0254	0.0273	0.0291	0.0309	0.0327	0.0344	0.0362	0.0379	0.0396	0.0413	0.0429	0.0444	0.0459	0.0472	0.0482	0.0487		
0.15	0.0249	0.0267	0.0285	0.0303	0.0321	0.0338	0.0355	0.0372	0.0389	0.0405	0.0421	0.0435	0.0448	0.0459	0.0465			
0.2	0.0243	0.0261	0.0279	0.0297	0.0314	0.0331	0.0348	0.0365	0.0381	0.0397	0.0411	0.0425	0.0436	0.0442				
0.25	0.0237	0.0255	0.0273	0.0290	0.0307	0.0324	0.0341	0.0357	0.0373	0.0388	0.0401	0.0412	0.0419					
0.3	0.0231	0.0249	0.0266	0.0283	0.0300	0.0317	0.0333	0.0349	0.0364	0.0377	0.0388	0.0396						
0.35	0.0225	0.0242	0.0259	0.0276	0.0293	0.0309	0.0325	0.0340	0.0353	0.0365	0.0373							
0.4	0.0218	0.0235	0.0252	0.0269	0.0285	0.0301	0.0315	0.0329	0.0341	0.0349	0.0350							
0.45	0.0211	0.0228	0.0245	0.0261	0.0276	0.0291	0.0305	0.0317	0.0325	0.0327								
0.5	0.0204	0.0221	0.0237	0.0252	0.0267	0.0281	0.0292	0.0301	0.0304									
0.55	0.0196	0.0212	0.0228	0.0243	0.0256	0.0268	0.0277	0.0280										
0.6	0.0188	0.0204	0.0218	0.0232	0.0244	0.0253	0.0256											
0.65	0.0179	0.0194	0.0207	0.0219	0.0228	0.0231												
0.7	0.0169	0.0183	0.0195	0.0204	0.0206													
0.75	0.0158	0.0170	0.0179	0.0181														
0.8	0.0145	0.0154	0.0155															
0.85	0.0129	0.0130																
0.9	0.0104																	